The finite element method can also be applied to the solution of many non-linear problems encountered in applications. For static non-linear problems, the FEM yields a system of non-linear algebraic equations. The system is usually solved using iterative methods (e.g. Newton-Raphson). However, convergence is not necessarily guaranteed and in some cases it may be difficult to obtain. Transient problems can be even more challenging as the solution of a non-linear system of algebraic equations is required at each time step. In this homework project you will apply the finite element method to the solution of a few selected non-linear problems.

1.- A cylindrical sample (radius 0.0125 m, length 0.1 m) of HY-100 steel is pulled uniaxially using a tensile testing machine. When the stress in the sample reaches about 790 MPa, it yields and begins to deform plastically. To account for yielding, assume that the stress-strain curve in the plastic domain is linear with a modulus 100 times smaller than the elastic modulus. The goal is to estimate the strain and stress fields inside the bar as a function of the applied load. Construct a finite element model of the system. Verify and validate your model.

2.- Hot water (T=343 K) enters a pipe (radius 0.0125 m, length 0.1 m) whose wall is maintained at constant temperature T=298 K. As the water flows inside the pipe, it cools. The goal is to estimate the temperature field inside the flowing water. Verify and validate your model.

3.- Solidification of a molten metal in a mold. Molten copper rapidly fills an iron mold of constant square cross section. The mold is initially at room temperature and copper is initially at 1400 K. The cross section of the mold opening is 0.05 by 0.05 m and the mold wall thickness is 0.1 m. To account for the latent heat of solidification $L$, assume that the specific heat is a function of temperature such that within the melting range between Delta $T_s = 1360-1350 = 10$, the “effective specific heat” $C_{p,eff} = L/Delta T_s$. The goal is to estimate the rate of solidification of copper inside the mold. Construct a finite element model of the system. Verify and validate your model.