A Finite Element Method Study of Coefficients of Restitution in Golf Driver Clubfaces

by

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NOMENCLATURE

\[ \begin{align*}
e & \quad \text{Coefficient of Restitution (-)} \\
V_{in} & \quad \text{Initial Velocity (mph)} \\
V_{out} & \quad \text{Rebound Velocity (mph)} \\
m_1 & \quad \text{One-Dimensional Model Ball Cover Mass (kg)} \\
m_2 & \quad \text{One-Dimensional Model Ball Core Mass (kg)} \\
m_t & \quad \text{One-Dimensional Model Total Ball Mass (kg)} \\
k & \quad \text{One-Dimensional Model Spring Constant (kg/s}^2\text{)} \\
c & \quad \text{One-Dimensional Model Damper Constant (kg/s)} \\
x & \quad \text{One-Dimensional Model Displacement (m)} \\
m_0 & \quad \text{One-Dimensional Model Clubhead Mass (kg)} \\
M_f & \quad \text{One-Dimensional Model Clubface Mass (kg)} \\
\rho & \quad \text{Density (kg/m}^3\text{)} \\
E & \quad \text{Modulus of Elasticity (MPa)} \\
E_0 & \quad \text{Initial Modulus of Elasticity (MPa)} \\
\nu & \quad \text{Poisson’s Ratio (-)} \\
\sigma_{yield} & \quad \text{Yield Stress (MPa)} \\
C_{01} & \quad \text{Mooney-Rivlin Coefficient (MPa)} \\
C_{10} & \quad \text{Mooney-Rivlin Coefficient (MPa)} \\
D & \quad \text{Diameter (mm)} \\
V_y & \quad \text{Y-Component of Velocity (mph)} \\
t_{cf} & \quad \text{Clubface Thickness (in)} \\
M & \quad \text{Clubface Mass (g)} \\
m & \quad \text{Ball Mass (g)}
\end{align*} \]

* Note Clubface Thickness and Velocities are given in inches and miles per hour. This is to allow for more easily relatable units. The rest are given in SI units for ANSYS-ED analysis purposes.
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I would like to thank the faculty and staff of RPI at Hartford and Electric Boat for helping me achieve my Masters degree. Linda Knaack at the Cole Library for her help in gathering resource materials. Matthew Pringle of the USGA for his assistance in understanding and providing the new and old methods for the testing of golf clubs. Most of all, I would like to thank professor Ken Brown for his extensive knowledge of the software used and his willingness to answer my many questions.
ABSTRACT

This project analyzes the effect the golf driver’s clubface thickness has on resulting coefficient of restitution (COR) of a golf ball impact for different swing speeds. Utilizing the finite element software packages ANSYS-ED and LS-DYNA, a representative axisymmetric model of a clubface and golf ball was created and the impact of a golf swing was simulated. The club was represented as a titanium alloy plate previously used by the USGA. The ball was modeled to mimic a real 3-piece golf ball with a mixture of butadiene and ionmer materials utilizing Mooney-Rivlin hyperelastic properties. The analysis was run for varying club speeds from 90 mph to 110 mph and varying clubface thicknesses from 0.075 inches to 0.1375 inches. As clubface thickness was increased, rebound ball velocities also increased for all initial swing speeds. Results show higher than theoretically possible values (0.930) for COR in all test cases. The most likely cause of this issue is the inaccuracy of the material modeling of the golf ball. Despite the high values, general trends were still observed. The highest COR value was 0.974 with a combination of a 90 mph swing speed and 0.1125” clubface thickness. Increasing or decreasing the clubface thickness from this point only further reduced COR for all swing speeds. Therefore, having a very thin clubface does not necessarily result in the highest achievable COR, but rather there is an optimized clubface thickness. The optimized clubface balances rebound velocity with club mass to produce the highest COR possible.
1. Introduction

1.1 Background

Golf clubs have seen dramatic improvements over recent years with the advancement in materials and manufacturing technologies. The number 1 wood, commonly known as the driver, is no longer made with a combination of hardwoods or steels. Instead they utilize the weight and strength benefits of graphite composites for shafts and titanium alloys for the clubhead. Even today, as the equipment advances, rules are often imposed or updated to keep the game fair and balanced. In 1984, around the time titanium drivers were in the experimental stages, the United States Golf Association (USGA) implemented Appendix II, (5a.) to the USGA Rules of Golf, which states;

“The material and construction of, or any treatment to, the face or clubhead shall not have the effect at impact of a spring, or impart significantly more spin to the ball than a standard steel face, or have any other effect which would unduly influence the movement of the ball.”

There was one underlying reason for this rule addition. The thin clubfaces achievable with hollow titanium drivers demonstrated a unique phenomenon during impact with a golf ball. It was observed [1] that the clubface would deform elastically upon impact, and return to its original geometry after, much like a trampoline. Thus the term “trampoline effect” was coined. In the older solid wood and steel clubs, the face was almost completely rigid and most of the energy loss during impact (friction, heat, sound, etc.) was occurring in the golf ball [3]. With the trampoline effect, the clubface absorbs some of the energy normally lost at impact and then returns it back to the ball, like a spring.

1.2 Coefficient of Restitution

The coefficient of restitution, or COR, measures the velocity ratio during an impact event. COR is represented as a ratio, with a value from 0 to 1. A COR with a value of 0 represents a perfectly inelastic collision. An example of this would be two bodies coming to a complete stop during impact. A COR with a value of 1 portrays a perfectly elastic collision, in which no energy is lost during impact. Usually COR is measured in terms of pre and post impact velocities. For example, take a ball hitting a rigid plate
with an initial velocity of 100 mph, and a post impact velocity of 80 mph. This impact has a COR of 0.80 or 80% of the ball’s energy was returned to the ball after impact. Equation 1 below shows the most basic COR formulation [10].

\[ COR = e = \frac{V_{\text{out}}}{V_{\text{in}}} \]  

(1)

1.2.1 History of COR in Golf

The USGA limited the COR in drivers to 0.830 in 1998, while the other governing body for golf outside the US, the Royal & Ancient Golf Club of St. Andrews (R&A), did not impose a limit on COR at that time. This caused confusion as to what drivers were allowed during play, especially for international events. In May of 2002, talks between the two governing bodies unveiled a proposal to establish the limit from 0.830 to 0.860 to create some uniformity around the globe [5]. Some manufacturers began producing drivers that exceeded the 0.830 limit in July of 2002, even though the rule was not yet official. This caused significant turbulence when the USGA decided to maintain its limit of 0.830, and the R&A decided it would enact the same limit beginning in 2008. These new drivers with nonconforming COR, or “hot” drivers, were deemed illegal for all tournament play and handicapped based rounds [5].

1.2.2 Testing COR for Drivers

With the introduction of these limits, The USGA needed a test procedure to measure a driver’s COR. Originally, a ball was fired by air cannon into a specimen and pre and post impact velocities were compared to find COR [7]. This process took a significant amount of time to perform when considering the set up (scribing clubs, finding center of gravity, etc…) and the controls associated with the golf balls used in the test. Today, the COR is measured using the “Characteristic Time” test, which consists of a steel ball with sensors on a pendulum being swung into a clubface [6]. The length of time the steel ball is in contact with the face determines the COR. For the purpose of this analysis, the model will refer back to the original air cannon testing procedure in [7].
1.3 Problem Description

The goal of this project was to analyze the trampoline effect and COR in drivers using finite element software. The changes in COR will be noted while varying the clubface thickness and initial velocities. For reference, at a swing speed of 100 mph a club at the COR limit of 0.830 will hit the ball around 10-15 yards more than a club with a rigid clubface (COR of about 0.770). At 100 mph, the average difference in performance between 0.860 and 0.830 COR is about 5.6 yards more [6]. At slower swing speeds the difference is even less. Thus, the trampoline effect is said to only benefits better players with higher swing speeds.

In previous works [3] the collision of a golf ball and club is said to lose more energy in deformation and recovery for a rigid clubface, yielding the smaller COR. It is suggested that having a thinner clubface will allow for reduced energy loss and increase the speed imparted back into the ball. That is, if less energy is lost in the clubface’s deformation than in that part of the ball’s deformation which it replaces [3][4].

The dynamics of the shaft are not included in this analysis based on the impact time being so short, less than 0.5 ms [2]. Stated in the [2], the collision between the ball and club is indeed initiated by the golfer swinging the club. The position and orientation of the clubface can and will create significantly different results, hence the need for a simplified model. During the short duration of impact, neither the golfer nor shaft can react to the dynamics of impact, limiting the golfers influence to be represented by the initial conditions of the model just before the moment of impact.

The model uses the geometry and properties of a representative clubface originally used to calibrate the test rig in the outdated USGA air cannon COR test [7]. It represents a typical weight of around 190g and utilizes a widely used titanium alloy found in many drivers. The golf ball will be based off of information from previous articles [1][2], and consist of three layers. The ball will have a hard ionomer resin cover and a soft, rubber like polybutadiene core and mantle, shown in Figure 1.
For the preprocessing and creation of the clubface model, ANSYS-ED will be used. The associated simulation of the impact will be done using LS-DYNA. LS-DYNA uses an explicit solver to simulate such impacts. An explicit solver predicts the state variables of a dynamic system at time $t + dt$ using the state variable of the system at time $t$.

1.4 Previous Work

While no articles could be found that specifically investigated the trampoline effect, many articles have been written on the subject of finite element analysis on golf ball-club interactions [1][2]. The majority of articles found related to the topic of properly modeling the dynamics of golf balls at impact. The unique inelastic and hyper-elastic properties of the golf ball required precise element, material, and mesh choices. Many studied the validity of the FE model by comparing it to results of a simplified physical test. One article that is consistently referenced by later works [1] suggested the idea of Mooney-Rivlin hyperelastic and viscoelastic modeling for finite element analysis. This article went into great detail on proper material modeling and the effects on the finite element results. It further compared these results to physical data and also validated the method by applying it to another type of golf ball.
Another published article [2] used LS-DYNA to optimize clubface design using the mechanical impedance matching theory. This theory suggests that the most efficient impact occurs when the natural frequencies of the colliding bodies are equal. Modal analysis was used to find the natural frequencies of the clubface and ball. The ball showed significantly lower frequencies than the clubface. Design modifications were implemented into the clubface in an attempt to lower the club’s frequency to better match that of the ball. The resulting clubface design resulted in better performance on the rebound velocity of the golf ball.
2. Methodology

2.1 Theory

2.1.1 One-Dimensional Impact Modeling

In previous works [3][4] the development of a one-dimensional model was created to simulate golf club and ball impacts. This is the most simplified form of modeling and represents the basics of golf collision analyses. A model must represent not only the ball, but the ball and club interaction. With the imposition of limits on COR, an accurate one dimensional model is useful for predicting results without performing direct testing.

In the most basic form, the ball impacts a massive stationary block with near normal (90 degrees) impact. The model is defined as having two masses connected by a non-linear spring in parallel with a non-linear damper [3]. Linear springs and dampers alone cannot reproduce the way in which coefficient of restitution and duration of contact vary with impact speed [2]. The ball is identified having two parts, a core and a cover with masses $m_1$ and $m_2$, respectively (total mass = $m_1 + m_2$). During impact, the forces provided by the spring and damper act on the core only, yielding the equation found below.

$$m_2 \ddot{x} = -kx|\dot{x}|^a - cx|x|^b$$  \hspace{1cm} (2)

Where the single and double dots represent the first and second derivatives of displacement with respect to time (velocity and acceleration). Figure 2 shows this simple model.

![Figure 2: Basic One-Dimensional Model [4]](image)
To model specific balls, the values for \( k, c, a, b \) and \( m_2 \) in equation 2 must be found. This is done by taking direct measurements from physical testing and adjusting the parameters to fit the outcomes.

To recreate the effects of a non-rigid clubface, a more complete model is needed. In Figure 3 below, a model is proposed with a flexible clubface with linear spring and dampening coefficients. The ball is still considered non-linear.

![Figure 3: One-Dimensional Model Representing a Flexible Clubface [4]](image)

In this model, the clubhead has two masses, where \( M_f \) is considered the effective “face” mass, and \( m_0 \) is the remainder. During impact, the clubface and ball cover masses are assumed to be a single unit defined as \( m_1 \). The three oscillating masses have the following equations of motion.

\[
\begin{align*}
    m_0\ddot{x}_0 &= k_1(x_1 - x_0) + c_1(\dot{x}_1 - \dot{x}_0) \\
    m_1\ddot{x}_1 &= -k_1(x_1 - x_0) - c_1(\dot{x}_1 - \dot{x}_0) + k_2(x_2 - x_1)\|x_2 - x_1\|^a + c_2(\dot{x}_2 - \dot{x}_1)\|x_2 - x_1\|^b \\
    m_2\ddot{x}_2 &= -k_2(x_2 - x_1)\|x_2 - x_1\|^a - c_2(\dot{x}_2 - \dot{x}_1)\|x_2 - x_1\|^b
\end{align*}
\]
Using results from experimentally testing golf balls and taking some basic assumptions for the mass of the clubface and impact speed, the following figure is presented [4].

![Figure 4: COR vs. Clubface Stiffness for One-Dimensional Model [4]](image)

As seen in Figure 4, the trend is in agreement with what is expected for a flexible clubface and this represents the most basic modeling of the trampoline effect. As the stiffness of the face is decreased, the COR increases. There also appears to be an optimal face stiffness. This project attempted to recreate these results by “adjusting” face stiffness by varying the clubface thickness.

### 2.2 Finite Element Model Development

In order to limit the size of the model in ANSYS-ED and reduce the simulation run time, a 2-d axisymmetric model was made instead of a 3-d model. A cross section of the titanium plate and 3-piece golf ball was produced with symmetry about the y-axis. The representative plate is made out of a titanium alloy to a specified weight of 190g. Its material properties are shown below in Table 1 and the geometry is shown in Figure 5.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$E$ (MPa)</th>
<th>$\nu$ (-)</th>
<th>$\sigma_{yield}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-Al6V4</td>
<td>4,500</td>
<td>113,000</td>
<td>0.34</td>
<td>1,100</td>
</tr>
</tbody>
</table>
The 3-piece golf ball was modeled using information from [2] that previously developed finite element golf balls for use with LS-DYNA. $C_{01}$ and $C_{10}$ are the Mooney-Rivlin coefficients and $D$ is the diameter of the separate layers of the ball components. The ball properties are given in Table 2.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$E_0$ (MPa)</th>
<th>$\nu$</th>
<th>$C_{01}$ (MPa)</th>
<th>$C_{10}$ (MPa)</th>
<th>$D$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butadiene (core)</td>
<td>1,150</td>
<td>50</td>
<td>0.49</td>
<td>10.2</td>
<td>1.46</td>
<td>35.4</td>
</tr>
<tr>
<td>Butadiene (mantel)</td>
<td>1,150</td>
<td>25</td>
<td>0.49</td>
<td>5.83</td>
<td>0.83</td>
<td>38.8</td>
</tr>
<tr>
<td>Ionomer (cover)</td>
<td>950</td>
<td>400</td>
<td>0.45</td>
<td>43.8</td>
<td>6.25</td>
<td>42.8</td>
</tr>
</tbody>
</table>

The titanium plate was modeled as rectangular areas merged together using the merge tool within ANSYS-ED. The small chamfers and fillets shown in Figure 5 are ignored to simplify the model and to eliminate complicated meshing. The elements chosen are LS-DYNA explicit Solid 162 elements. The elements can either be quadrilateral or triangular in shape. However, quad elements produce much more accurate results than the notoriously stiff tri elements and are therefore preferred. The 3-piece ball was
modeled as concentric axisymmetric semicircles about the y-axis. The constitutive behavior of the ball was represented by the Mooney-Rivlin rubber elastic model characteristics as in [1][2]. It was also modeled using the Solid 162 elements. The separate areas for the different layers are joined together using the merge tool. This is to ensure that all the components of the ball move as one during the impact event. The different colors represent the different areas within the model. The model cross section is shown in Figure 6.

![Figure 6: Cross Section of Areas Used for Axisymmetric Modeling](image)

After the areas are defined, material properties of the three layers of the golf ball and the titanium plate are applied. Under the mesh tool within ANSYS-ED, the number of element division on a line can be specified, and were used to create a suitably accurate mesh. The mesh was then generated and checked for any errors or use of tri elements. If errors were found, the element divisions were altered until a mesh consisting of pure quad elements was achieved. The final mesh was produced exhibiting 1054 nodes and 949 elements. These numbers stay constant for all test cases because the element divisions on each line were not altered when altering plate thickness. The mesh is shown in Figure 7, with the different colors representing the material properties specified.
Figure 7: Materials Defined and Mesh Created

Using the axisymmetric expansion option within ANSYS-ED, the cross section can be revolved around the y-axis a specified number of degrees for visualization purposes. This helps gain a better grasp of what the 2-d axisymmetric model is trying to represent in 3-d space. Below in Figure 8, the cross section is expanded 270 degrees.

Figure 8: Cross Section of Model Expanded 270 Degrees
2.3 LS-DYNA Contact Analysis & Initial Conditions

The ball is initialized to have a negative y-velocity component. The same speed is used for six varying clubface thicknesses and then altered. The speeds used for the initial y velocity will be representative to a range of typical male swing speeds of 110, 100, and 90 mph [9]. The speeds and thicknesses were converted to SI units (m/s and m) for the analysis to keep consistent units in ANSYS-ED. The initial velocity was applied to the ball by selecting all the nodes of the ball and introducing a y-velocity component for them in the LS-DYNA initial velocity menu in the preprocessor. The analysis then investigated the clubface thicknesses and speeds shown in Table 3 below.

<table>
<thead>
<tr>
<th>$V_y$ (mph)</th>
<th>$t_{cf}$ (in)</th>
<th>$V_y$ (mph)</th>
<th>$t_{cf}$ (in)</th>
<th>$V_y$ (mph)</th>
<th>$t_{cf}$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-110</td>
<td>0.1375</td>
<td>0.1375</td>
<td>0.1375</td>
<td>0.1375</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.1250</td>
<td>0.1250</td>
<td>0.1250</td>
<td>0.1250</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.1125</td>
<td>0.1125</td>
<td>0.1125</td>
<td>0.1125</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0875</td>
<td>0.0875</td>
<td>0.0875</td>
<td>0.0875</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0750</td>
<td>0.0750</td>
<td>0.0750</td>
<td>0.0750</td>
<td></td>
</tr>
</tbody>
</table>

For the analysis, a solution time of 0.005 seconds was specified with 100 result “snapshots”. The impact occurs in less than 0.001s, but in order to accurately reproduce the USGA test that measure the velocity a few inches past the point of impact, the time is extended to recreate this scenario. The increased length in solution time allows the ball to “settle out” after impact and reach a steady state velocity. Using the output options in ANSYS-ED, an ASCII output file can be produced which gives the material energies for the entire solution. This creates a file called matsum that displays the averaged nodal velocities of each model part (core, mantel, cover, and plate) for all time steps of the solution. At time 0.005 seconds, the velocity of the core will be chosen as the representative velocity for the entire ball. This value was validated and then used for the comparisons.
The mass of each part will play an important role in determining accurate COR. ANSYS-ED/LS-DYNA automatically produces a file called d3hsp that will give the mass of each part per radian. To get the mass of the entire model one must multiply by $2\pi$.

The clubface is represented to be in free space. The USGA sets up their air cannon rig such that when the ball impacts the plate, the plate is not constrained in any direction. This allows for the influence to be purely based on the mass of the plate and ball. Therefore, there are no boundary constraints imposed on the plate or ball, and the only initial condition is the velocity placed on the ball in the negative y direction. There are however, axisymmetric properties on the entire model, which will limit the model to have no tangential motion.

### 2.4 Post Processing

Once the final velocity and masses are known, the resulting COR was found using the equation given from the USGA test procedure [7]. Since the impact can be changed dramatically depending on the mass of either the ball or club, both are taken into account. Equation 4 below shows the appropriate expression for COR taken from the USGA’s test procedure.

$$e = \left[\left(\frac{V_{\text{out}}}{V_{\text{in}}}ight)(M + m) + m\right]/M$$  \hspace{1cm} (4)

Where $M$ is the mass of the plate and $m$ is the mass of the ball. The mass of the plate, M, will always be multiplied by a fraction in the numerator, while in the denominator, it will always stay to the first power. This is where the influence of the plate’s mass will come in the most, for a constant velocity ratio and as only plate mass decreases, the COR increases slightly. However, a change in plate mass may also alter the velocity ratio, not necessarily leading to increased COR.
With the final velocities and COR found, Microsoft Excel was used to plot comparative charts of velocity and COR versus the clubface thickness and swing speed according to Table 3. Excel was also used to validate the length of time the solution runs, 0.005 seconds. This was done by examining a test case’s velocity vs. time graph to ensure that the solution time allowed stabilized rebound velocities.
3. Results

3.1 Rebound Velocity Results

Before post processing began, the solution time needed to be validated. Taking the clubface thickness of 0.125” and a swing speed of 110 mph as an example, the ball velocity (taken from the matsum file) was plotted vs. time to see if the 0.005 second solution time was adequate. The results are shown in Figure 9 below.

![Ball Velocity vs. Time for 0.125" Clubface and 110 MPH Swing Speed](image)

As seen above, the velocity of the ball increases sharply from the initial 110 mph swing speed immediately after impact and remains stable throughout. Therefore, it can be assumed to be well settled at a time of 0.005s. Also at this time ball is found to be approximately 6” away from the plate by multiplying speed and time. This accurately represents the USGA air cannon test in which the velocity was measured a few inches past the plate after impact, as discussed in section 2.3.
Taking the post impact velocity at the snapshot time of 0.005 seconds from the matsum file, the following results were found. Recall that the averaged nodal velocity of the core was used as the representative velocity for the entire ball. The velocity components of the ball’s mantle and cover are considered to be non-contributing. The velocities for each clubface thickness and swing speed are shown in Table 4.

### Table 4: Rebound Velocity Results

<table>
<thead>
<tr>
<th>Clubface Thickness</th>
<th>0.1375”</th>
<th>0.1250”</th>
<th>0.1125”</th>
<th>0.1000”</th>
<th>0.0875”</th>
<th>0.0750”</th>
</tr>
</thead>
<tbody>
<tr>
<td>V(_{in}) (mph)</td>
<td>V(_{out}) (mph)</td>
<td>V(_{in}) (mph)</td>
<td>V(_{out}) (mph)</td>
<td>V(_{in}) (mph)</td>
<td>V(_{out}) (mph)</td>
<td>V(_{in}) (mph)</td>
</tr>
<tr>
<td>110</td>
<td>65.348</td>
<td>110</td>
<td>64.815</td>
<td>110</td>
<td>62.106</td>
<td>110</td>
</tr>
<tr>
<td>100</td>
<td>59.829</td>
<td>100</td>
<td>59.332</td>
<td>100</td>
<td>57.138</td>
<td>100</td>
</tr>
<tr>
<td>90</td>
<td>54.416</td>
<td>90</td>
<td>53.868</td>
<td>90</td>
<td>51.998</td>
<td>90</td>
</tr>
</tbody>
</table>
The results in Table 4 are shown graphically in Figure 10. The different color curves represent the different swing speeds.

![Rebound Velocity vs. Clubface Thickness for Different Swing Speeds](image)

Figure 10: Rebound Velocity vs. Clubface Thickness for Different Swing Speeds

### 3.2 Coefficient of Restitution Results

The first step in calculating COR is indentifying the mass of the ball and plate from the d3hsp file for use in Equation 2. The mass of the ball stays constant for each analysis, as the material properties and dimensions do not change. As one makes the plate thinner or thicker, the mass of the plate changes an amount of roughly 6.5 grams for every 0.0125” decrease in thickness. Table 5 shows the calculated mass of the ball and the plate for each thickness addressed.
Table 5: Calculated Masses for 3-Piece Golf Ball and Titanium Plate

<table>
<thead>
<tr>
<th>Component</th>
<th>Mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete Ball Mass</td>
<td>$m = 45.03$</td>
</tr>
<tr>
<td>0.1375” Clubface Thickness Plate Mass</td>
<td>$M = 200.58$</td>
</tr>
<tr>
<td>0.1250” Clubface Thickness Plate Mass</td>
<td>$M = 194.10$</td>
</tr>
<tr>
<td>0.1125” Clubface Thickness Plate Mass</td>
<td>$M = 187.55$</td>
</tr>
<tr>
<td>0.1000” Clubface Thickness Plate Mass</td>
<td>$M = 181.04$</td>
</tr>
<tr>
<td>0.0875” Clubface Thickness Plate Mass</td>
<td>$M = 174.52$</td>
</tr>
<tr>
<td>0.0750” Clubface Thickness Plate Mass</td>
<td>$M = 168.01$</td>
</tr>
</tbody>
</table>

Substituting the masses from Table 5 and final velocities found in Table 4 into Equation 4, the values of COR can be found. These results are shown in Table 6 below.

Table 6: Coefficient of Restitution Results

<table>
<thead>
<tr>
<th>0.1375” Clubface Thickness</th>
<th>0.1250” Clubface Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{in}$ (mph)</td>
<td>COR (-)</td>
</tr>
<tr>
<td>110</td>
<td>0.952</td>
</tr>
<tr>
<td>100</td>
<td>0.957</td>
</tr>
<tr>
<td>90</td>
<td>0.965</td>
</tr>
<tr>
<td>0.1125” Clubface Thickness</td>
<td>0.1000” Clubface Thickness</td>
</tr>
<tr>
<td>$V_{in}$ (mph)</td>
<td>COR (-)</td>
</tr>
<tr>
<td>110</td>
<td>0.959</td>
</tr>
<tr>
<td>100</td>
<td>0.964</td>
</tr>
<tr>
<td>90</td>
<td>0.974</td>
</tr>
<tr>
<td>0.0875” Clubface Thickness</td>
<td>0.0750” Clubface Thickness</td>
</tr>
<tr>
<td>$V_{in}$ (mph)</td>
<td>COR (-)</td>
</tr>
<tr>
<td>110</td>
<td>0.941</td>
</tr>
<tr>
<td>100</td>
<td>0.950</td>
</tr>
<tr>
<td>90</td>
<td>0.960</td>
</tr>
</tbody>
</table>
The results of Table 6 are shown graphically in Figure 11. The different color curves represent the different swings speeds analyzed.

![COR vs. Clubface Thickness for Different Swing Speeds](image-url)

**Figure 11: COR vs. Clubface Thickness for Different Swing Speeds**

### 3.3 Discussion

#### 3.3.1 Rebound Velocities

From Tables 4 and Figure 10 it can be seen that the final velocity of the ball decreased as clubface thickness was decreased. One may think this will automatically result in decreased COR. However, COR takes into account the mass of the ball and plate, and the velocity ratio is just a single component of the formulation. COR results are discussed in section 3.3.3. For each clubface thickness analyzed, the higher the swing speed, the greater the return velocities. The difference in final velocity between each swing speed is less for the thinnest clubface thickness 0.0750” and greatest at the thickest clubface thickness of 0.1375”.
3.3.2 Ball and Plate Masses

Table 5 shows the calculated masses of the ball and plate from the ANSYS-ED d3hsp file. Using the USGA COR test’s mean masses for the balls and test plates as a reference [7], the results show that these masses are very close to actual values, and therefore validate the material densities and dimensional specifications used. Table 7 below shows the comparison between calculated and actual masses from the USGA test for a standard ball and 0.125” plate.

**Table 7: Comparison of Actual vs. Calculated Masses**

<table>
<thead>
<tr>
<th></th>
<th>Actual (USGA)</th>
<th>Calculated (ANSYS)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-Piece Golf Ball</td>
<td>( m = 45.40 ) g</td>
<td>( m = 45.03 ) g</td>
<td>0.81</td>
</tr>
<tr>
<td>0.125” Titanium Plate</td>
<td>( M = 190.00 ) g</td>
<td>( M = 194.10 ) g</td>
<td>2.16</td>
</tr>
</tbody>
</table>

3.3.3 Coefficient of Restitution

Although the masses of the ball and plate agree, the COR for all test scenarios are generally too high. The expected mean COR of the 0.125” plate from the USGA test procedure is 0.822 at a swing speed of 110 mph [7]. Table 8 below shows the comparison of the calculated vs. the expected COR for this scenario.

**Table 8: Comparison of Actual vs. Calculated COR**

<table>
<thead>
<tr>
<th></th>
<th>Actual (USGA)</th>
<th>Calculated (ANSYS)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125” Titanium Plate</td>
<td>( e = 0.822 )</td>
<td>( e = 0.958 )</td>
<td>14.20</td>
</tr>
</tbody>
</table>

The high values for COR may be due to inadequate Mooney-Rivlin hyperelastic ball properties. The coefficients for Mooney-Rivlin were taken from a previous article [2] where the goal was not to analyze COR, but rather optimize a clubface, so 100% ball model accuracy may not have been a concern. This is similar to our analysis, where the primary concentration of the article is in the clubface and not the ball. Even with the high values for COR, the general trends can be observed. As suggested in [1],
viscoelastic material properties were proposed in the ball, which was neglected in this project. This is the most likely reason for the high COR, as we are observing near fully elastic behavior by neglecting the true reactions in the ball during impact.

3.3.3.1 COR vs. Swing Speed

Unlike the final velocities, as swing speed was increased, the COR decreased, as shown in Figure 11. This does not agree with the previous argument that a high COR benefited players with higher swing speeds more than players with a lower swing speed. However, similar results were obtained in [4]. Higher swing speeds do indeed produce higher final velocities, but the efficiencies of these impacts decreases as shown with the lower CORs.

3.3.3.2 COR vs. Clubface Thickness

This is the primary focus of this project. As observed in Table 6 and Figure 11, it appears that there is an optimized clubface thickness to achieve as high as possible COR for all swing speeds. Based on the data points taken, a clubface thickness of 0.1125” produced the highest COR for each swing speed. The general trend of peaking COR is in agreement with results published in [4] (see Figure 3).

As the clubface thickness decreases or increases from 0.1125”, the COR declines. It is noted that as the face gets thinner, the COR decreases at a faster rate than it does as the face gets thicker. This goes against the notion that the thinner the clubface, the higher the COR. According to Equation 4, if you decrease just plate thickness (and therefore mass) the COR would increase. However, based on section 3.1, the decrease in plate thickness also decreases the output velocity, which lowers COR. It appears that there needs to be a balance of clubface mass and thickness to achieve the most desirable effect.
4. Conclusions

This project used the finite element method to model a golf ball and clubface to analyze the effects of face thickness on COR. A noted issue with this model was, that as the thickness of the face decreased, the mass also decreased, reducing the rebound velocity as shown in Figure 10. This caused a negative effect on achieving the highest COR as can be seen by Equation 4. The equation has the rebound velocity in the numerator to the first power, decreasing this also decreases the overall COR. While we were able to discover an optimal face thickness and observe some notable trends for our representative model, the true goal of understanding the trampoline effect was not achieved. To do so, a model would have to be created using a complete clubhead body. This would help maintain the majority of the mass as face thickness decreased. The effects of solely decreasing clubface thickness while maintaining mass may produce the results that were expected for this project. An alternative for the model used in this project would have been to take the mass lost from decreasing the clubface thickness, and adding it back into the surrounding ring of the representative club. This would effectively do the same thing; decrease clubface thickness while maintaining mass.

A suggestion for improvement would be to use the full version of ANSYS, as the educational version, ANSYS-ED, limited the number of elements able to be utilized. For this project the max number of elements was used, and seemed to create a suitable mesh. If more elements were available, one could analyze the effects mesh size has on the results.

Another future topic would be to include the effects of viscoelasticity in the model of the ball, instead of just hyper elasticity. This was suggested in [1] and is believed to be the reason why the COR was higher than expected. More time working with ANSYS software packages would be needed to correctly understand and apply these material properties.
5. References


<http://www.wishongolf.com/faq_tech_answer.php?techKey=1>


<http://golf.about.com/od/golfterms/g/clubheadspeed.htm>


APPENDIX A: ANSYS-ED/LS-DYNA LOG FILE

* By changing the highlighted areas, the clubface thickness and initial velocity can be changed to recreate any of the analysis scenarios described in Table 3.

```plaintext
*/
/COM,
/COM, Preferences for GUI filtering have been set to display:
/COM, Structural with LS-DYNA Explicit
*/
/MP, DENS, 1, 4500
/MP, EX, 1, 113000e6
/MP, NUXY, 1, 0.34
/TB, PLAW, 1, 1,
/TBDAT, 1, 1100e6
/TBDAT, 2,
/TBDAT, 3,
/TBDAT, 4,
/TBDAT, 5,
/TBDAT, 6,
/TBDAT, 7,
/MP, DENS, 2, 2150
/MP, NUXY, 2, 0.49
/TB, MOONEY, 2, 0,
/TBDAT, 1, 1.46e6
/TBDAT, 2, 10.2e6
/MP, DENS, 3, 2150
/MP, NUXY, 3, 0.49
/TB, MOONEY, 3, 0,
/TBDAT, 1, 1.83e6
/TBDAT, 2, 5.83e6
/MP, DENS, 4, 1150
/MP, NUXY, 4, 0.45
/TB, MOONEY, 4, 0,
/TBDAT, 1, 6.25e6
/TBDAT, 2, 43.8e6
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/DIST, 1, 1.08222638492, 1
/REP, FAST
/DIST, 1, 1.08222638492, 1
/REP, FAST
/PCIRC, 0.177, -90, 90,
/PCIRC, 0.0194, -90, 90,
/PCIRC, 0.0214, -90, 90,
/AUTO, 1
/REP, FAST
SAVE
FLST, 3, 3, 5, ORDE, 2
FITEM, 3, 1
FITEM, 3, 3
AGEN, P51X, , 03, , 1
/USER, 1
/FOC, 1, 0.103186524437E-01, 0.196188591589E-01, 0.0000000000
/REPLO
/FOC, 1, 0.113355804604E-01, 0.246187552410E-01, 0.0000000000
/REPLO
```
/AUTO,1
/REP,FAST
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/PNUM,KP,1
/PNUM,LINER0
/PNUM,AREA,0
/PNUM,VOLU,0
/PNUM,NODE,0
/PNUM,TabN,0
/PNUM,SVAL,0
/NUMBER,0
*/
PNUM,ELEM,0
/REPL0T
*/
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RECTNG,0.0254,0.0381,0.004902,0.008077,
RECTNG,0.0381,0.0508,0.004902,0.008077,
NUMMRG,ALL,,1,LOW
APLOT
/AUTO,1
/REP,FAST
SAVE
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FITEM,5,5
FITEM,5,7
FITEM,5,9
FITEM,5,11
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LSEL,,P51X
CM_ Y1,LINE
CMSEL_ Y
*/
LESIZE_ Y1,,2,,1
*/
FLST,5,3,4,ORDE,3
FITEM,5,1
FITEM,5,4
FITEM,5,8
CM_ Y,LINE
LSEL,,P51X
CM_ Y1,LINE
CMSEL_ Y
*/
LESIZE_ Y1,,40,,1
*/
FLST,5,2,4,ORDE,2
FITEM,5,2
FITEM,5,3
CM_ Y,LINE
LSEL,,P51X
CM_ Y1,LINE
CMSEL_ Y
*/
LESIZE_ Y1,,20,,1
*/
FLST,5,5,4,ORDE,5
FITEM,5,10
FITEM,5,13
FITEM,5,15
FITEM,5,19
FITEM,5,23
CM_ Y,LINE
LSEL,,P51X
CM_ Y1,LINE
CMSEL_ Y
*/
LESIZE, Y1, 5, , , 1

FLST, 5, 2, 4, ORDE, 2
ITEM, 5, 2, 7
ITEM, 5, 2, 9
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LSEL, , , P51X
CM, Y1, LINE
CMSEL, Y

LESIZE, Y1, 6, , , 1

FLST, 5, 9, 4, ORDE, 9
ITEM, 5, 6
ITEM, 5, 12
ITEM, 5, 14
ITEM, 5, 16
ITEM, 5, 18
ITEM, 5, 20
ITEM, 5, 22
ITEM, 5, 24
ITEM, 5, 26
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LSEL, , , P51X
CM, Y1, LINE
CMSEL, Y

LESIZE, Y1, 12, , , 1

FLST, 5, 5, 5, ORDE, 2
ITEM, 5, 4
ITEM, 5, 8
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ASEL, , , P51X
CM, Y1, AREA
CMSEL, S, Y

CMSEL, S, Y1
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CMSEL, S, Y
CMDELE, Y
CMDELE, Y1

CM, Y, AREA
ASEL, , , 1
CM, Y1, AREA
CMSEL, S, Y

CMSEL, S, Y1
AATT, 2 , 1 , 0,
CMSEL, S, Y
CMDELE, Y
CMDELE, Y1

CM, Y, AREA
ASEL, , , 2
CM, Y1, AREA
CMSEL, S, Y

CMSEL, S, Y1
AATT, 3 , 1 , 0,
CMSEL, S, Y
CMDELE, Y
CMDELE, Y1

CM, Y, AREA
ASEL, , , 3
CM, Y1, AREA
CMSEL, S, Y


CMSEL,S,Y1
AATT,4,1,0,
CMSEL,S,Y
CMDELE,Y
CMDELE,Y1
*
MSHAPE,0,2D
MSHKEY,0
*
FLST,5,8,5,ORDE,2
FITEM,5,1
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ASEL,,P51X
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CHKMSH,'AREA'
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*
AMESH,Y1
*
CMDELE,Y
CMDELE,Y1
CMDELE,Y2
*
/PNUM,KP,0
/PNUM,LINE,0
/PNUM,AREA,0
/PNUM,VOLU,0
/PNUM,NODE,0
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/REPLOT
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/PNUM,AREA,1
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/PNUM,TABN,0
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/NUMBER,1
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/REPLOT
*
EPLOT
SAVE
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FITEM,5,1
FITEM,5,-682
NSEL,,P51X
CM,Ball,NODE
CMSEL,S,BALL
CMPLOT
/MREP,EPL0T
EPL0T
* EDCGEN,ASS2D,,0,0,0,0,,0,1000000
* EDVE,VELO,BALL,0,49.2,0,0,0,,
FINISH
/SOL
TIME,0.005,
* EOUT,MATSUM
ALLSEL,ALL
SAVE
/STATUS,SOLU
SOLVE
FINISH