> restart; E := 1e11: rho := 8000: g := 9.81: f := rho*g: W := 1e4: A := 0.01:
> x0 := 0: xL := 10.0:
> N := 4: h := (xL - x0)/N:
> del := -E*diff(u(x), x$2) = f:
> bca := u(x0) = 0: bcb := D(u)(xL) = W/E*A:
> uex := dsolve( {del, bca, bcb}, u(x));
> uex := u(x) = -981/2500000000*x^2 + 26487/2500000000*x

(1)

> plot(rhs(uex), x = x0..xL):
> sex := E*diff(rhs(uex), x):
> for i from 1 to N+1 do x[i] := h*(i-1) + x0 end do:
> for i from 1 to N do p[i] := piecewise(x[i-1] <= x < x[i], (x-x[i-1])/(x[i]-x[i-1]), x[i] <= x <= x[i] + 1, (x-x[i+1])/(x[i]-x[i+1]), 0) end do:
> p[N+1] := piecewise(x[N] <= x <= x[N+1], (x-x[N])/(x[N+1]-x[N]), 0):
> plot({p[index]$index=1..N+1}, x=x[1]..x[N+1]):
> uG := sum(a[n]*p[n], n=1..N+1):
> dutdx := diff(uG, x):
> for j from 1 to N+1 do vG[j] := p[j] end do:
> for j from 1 to N+1 do dvGdx[j] := diff(vG[j], x) end do:
> eq[1] := a[1] = 0:
> for j from 2 to N do eq[j] := int(E*dutdx*dvGdx[j], x=x[j-1]..x[j+1]) = int(f*vG[j], x=x[j-1]..x[j+1]) end do:
> eq[N+1] := evalf(subs(x=xL, vG[N+1])*E*(W/E*A)) + int(E*dutdx*dvGdx[N+1], x=x[N]..x[N+1]) = int(f*vG[N+1], x=x[N]..x[N+1]):
> for j from 1 to N+1 do eq[j] end do:
> s1 := solve( {eq[index]$index=1..N+1}, {a[index]$index=1..N+1}):
> assign(s1):
> plot({rhs(uex), uG}, x=x[1]..x[N+1]);
\begin{equation}
\texttt{evalf(int(abs(uG - rhs(uex)), x=x0..xL));}
\end{equation}

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\texttt{evalf(int(abs(uG - rhs(uex)), x=x0..xL));}
\end{equation}

\begin{equation}
\texttt{0.00004087500000}
\end{equation}

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\end{equation}

\begin{equation}
\texttt{sG := E * duGdx: plot( \{sG, sex\}, x=x0..xL);} \tag{2}
\end{equation}
> evalf(int(abs(sG - sex), x=x0..xL));

4.905000000 10^5