Consider the again boundary value problem
\[ - \frac{d^2 u}{dx^2} = 20x^3 \]
subject to
\[ u(0) = u(1) = 0 \]

a) Use the method of Galerkin without integration by parts on the left hand side and the following two functions
\[ \phi_1 = \sin(\pi x) \]
\[ \phi_2 = \sin(2\pi x) \]
to obtain an approximate solution to the problem. Evaluate the accuracy of the approximation.

b) Use the method of Galerkin with integration by parts on the left hand side and the following two functions
\[ \phi_1 = \sin(\pi x) \]
\[ \phi_2 = \sin(2\pi x) \]
to obtain an approximate solution to the problem. Evaluate the accuracy of the approximation.
Consider the boundary value problem

\[-\frac{d^2u}{dx^2} = 20x^3\]

subject to

\[u(0) = u(1) = 0\]

a) Consider the following function

\[\phi_2(x) = \begin{cases} 
2x, & 0 \leq x \leq \frac{1}{2} \\
2(1-x), & \frac{1}{2} \leq x \leq 1 
\end{cases}\]

and use the method of Galerkin with integration by parts on the left hand side to obtain an approximate solution to the problem. Evaluate the accuracy of the approximation. What happens if you do not use integration by parts on the left hand side?

b) Consider the following functions

\[\phi_2(x) = \begin{cases} 
3x, & 0 \leq x \leq \frac{1}{3} \\
3\left(\frac{2}{3} - x\right), & \frac{1}{3} \leq x \leq \frac{2}{3} \\
0, & \frac{2}{3} \leq x \leq 1 
\end{cases}\]

and

\[\phi_3(x) = \begin{cases} 
0, & 0 \leq x \leq \frac{1}{3} \\
3(x - \frac{1}{3}), & \frac{1}{3} \leq x \leq \frac{2}{3} \\
3(1-x), & \frac{2}{3} \leq x \leq 1 
\end{cases}\]

and use the method of Galerkin with integration by parts on the left hand side to obtain an approximate solution to the problem. Evaluate the accuracy of the approximation. What happens if you do not use integration by parts on the left hand side?

c) Find a new approximation using the Galerkin method using the three functions \(\phi_2(x), \phi_3(x), \phi_4(x)\) that would follow in the pattern above.