restart;
> p1 := sin(Pi·x); p2 := sin(2·Pi·x); p3 := sin(3·Pi·x); p4 := sin(4·Pi·x);
p1 := sin(π·x)
p2 := sin(2·π·x)
p3 := sin(3·π·x)
p4 := sin(4·π·x)

> uG := a1·p1 + a2·p2 + a3·p3 + a4·p4;
uG := a1 sin(π·x) + a2 sin(2·π·x) + a3 sin(3·π·x) + a4 sin(4·π·x)

> d2udx2 := diff(uG, x$2$);
d2udx2 := -a1 sin(π·x) π^2 - 4 a2 sin(2·π·x) π^2 - 9 a3 sin(3·π·x) π^2 - 16 a4 sin(4·π·x) π^2

> f := 1:
de1 := -diff(u(x), x$2$) = f; bca := u(0) = 0: bcb := u(1) = 0:
s1 := dsolve({de1, bca, bcb}, u(x)); assign(%): u(x);
a1 = 4 π^3, a2 = 0, a3 = 427 π^3, a4 = 0

> eq1 := int(-d2udx2·p1, x=0..1) = int(f·p1, x=0..1);
eq1 := 12 π^2 a1 = 2 π

> eq2 := int(-d2udx2·p2, x=0..1) = int(f·p2, x=0..1);
eq2 := 2 a2 π^2 = 0

> eq3 := int(-d2udx2·p3, x=0..1) = int(f·p3, x=0..1);
eq3 := 93 a3 π^2 = 23 π

> eq4 := int(-d2udx2·p4, x=0..1) = int(f·p4, x=0..1);
eq4 := 8 a4 π^2 = 0

> solve({eq1, eq2, eq3, eq4}, {a1, a2, a3, a4}); assign(%);
\{a1 = 4 \pi^3, a2 = 0, a3 = 427 \pi^3, a4 = 0\}

> uG;
4 sin(\pi·x) π^3 + 427 sin(3·\pi·x) \pi^3

> plot({u(x), uG}, x = 0..1);