Mathematical modelling of MSW incineration on a travelling bed

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Abstract

The rising popularity of incineration of municipal solid waste (MSW) calls for detailed mathematical modelling and understanding of the incineration process. In this paper, governing equations for mass, momentum and heat transfer for both solid and gaseous phases in a moving bed in a solid-waste incineration furnace are described and relevant sub-models are presented. The burning rates of volatile hydrocarbons in the moving bed of solids are limited not only by the reaction kinetics but also the mixing of the volatile fuels with the under-fire air. The mixing rate is averaged across a computation cell and correlated to a number of parameters including local void fraction of the bed, gas velocity and a length scale comparable to the particle size in the bed. A correlation equation is also included to calculate the mixing in the freeboard area immediately next to the bed surface. A small-scale fixed bed waste incinerator was built and test runs were made in which total mass loss from the bed, temperature and gas composition at different locations along the bed height were measured. A 2-D bed-modelling program (FLIC) was developed which incorporates the various sub-process models and solves the governing equations for both gases and solids. Thermal and chemical processes are mainly confined within a layer about 5–9 times in thickness of the averaged particle size in the burning bed. For a large part of the burning process, the total mass loss rate was constant until the solid waste was totally dried out and a period of highly rising CO emission followed. The maximum bed temperature was around 1200 K. The whole burning process ended within 60 min. Big fluctuations in species concentration were observed due to channelling and subsequent ‘catastrophic’ changes in the local bed conditions. Reasonably good agreement between modelling and measurements has been achieved. Yet the modelling work is complicated by the channelling phenomenon in the bed. Numerical simulations without consideration of the channelling effect produced very good agreement with experiments concerning the total mass loss, but significant discrepancy exists for temperature and gas composition profiles. Transient phenomena such as the breaking of waste particles and the “catastrophic” creation of new burning channels occurring during waste incineration is a vital area requiring further investigation at the fundamental level. The underlying theory of bed behaviour must be extended to include these transient events. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

The growing scarcity of dumping sites for municipal solid waste and the increasing environmental problems with landfill have led to more stringent regulations on and increasing cost of waste disposal in many countries. A lot of alternative methods have been proposed, and among them, waste incineration has been gaining more and more popularity by significantly reducing the volume of collected material into an inert residue and at the same time producing a combination of heat and electricity.

The first municipal waste incinerator was built in 1876 in the UK (Reimann and Hammerli, 1995). It featured a packed bed combustor which was operated in a discontinuous way. Although there have been many improvements in the furnace design ever since, packed bed combustion is still the most common method of incinerating municipal solid waste. A layer of solid particles are packed on a grate at the bottom of the incinerator and an air feed is supplied to the furnace. If also the solid material is supplied continuously, a moving bed incinerator results. Most of the combustible material in waste is burned in the bed and its immediately adjacent freeboard region, producing CO₂ and H₂O if sufficient air is supplied, or plus CO and unburned hydrocarbons if there is a lack of oxygen in bed, which are then burned in the over-bed region by adding secondary air. The solid products are bottom ash and fly ash.

Accurate modelling of the sophisticated thermal and chemical processes in packed-bed incineration of waste is a challenge to scientists, considering the wide variations in waste composition and the many different pollutants
they may be generating. Nevertheless, mathematical modelling is a necessity, and a comprehensive and advanced computer program needs to be developed which can predict not only the burn-out of various wastes but also the formation of major pollutants and toxic materials. The work presented in this paper is part of a continuous effort by the authors to achieve this goal. In this paper, governing equations for mass, momentum and heat transfer for both solid and gaseous phases in a moving packed bed of waste solids are described and relevant sub-models are presented. A 2-D interactive computer program has been developed to solve all the concerned equations numerically, and theoretical predictions were made of the burning of a high-moisture and high-volatile solid waste in a moving packed bed. Validation of the numerical simulation was achieved by building and firing a bench-top stationary incinerator. Total mass loss rate, temperature and gas compositions at different locations along the bed height as a function of time were measured and compared.

The early attempt to model the incineration processes in packed-beds of wastes dates back to at least 1970s (Essenhigh and Kuo, 1970). The past few years, however, have been seeing a dramatic increase in the modelling work thanks to the growing interests in the waste incineration technology during the last decade. Peters (1995) summarised the general governing equations for both gaseous and solid phases in a moving bed, and Goh et al. (1998, 1999) carried out numerical calculations in a stationary shrinking bed of solid waste material. Other recent important work include those by Switchenbank et al. (1997), Beckmann et al. (1997), Kremer et al. (1999), Larfeldt et al. (2000), Pattikangas et al. (2000) and Zakaria et al. (2000), among many others. Most of the past work, however, were more on the empirical side and usually have not been able to give the spatial details of the incineration processes within the packed-beds. The current paper works on a much more detailed scale. The whole bed and the freeboard area above are divided into many small volumes, the transport equations concerning the flow, heat transfer and combustion of the solid and gas phases are then discretised over these volumes or cells, and solved iteratively over the whole computation domain. The computation gives the results on the distributions of temperature, waste components, gas species and other properties both within the bed and in the freeboard space. Individual process rates such as moisture evaporation, volatile devolatilization and char burning etc. can easily be monitored and analysed. Other features of the program include estimation of the channelling effect and analysis of the transient effects of changing the waste input or other bed operating conditions.

The following sections introduce the development of the numerical program and the comparison of numerical predictions against the experimental measurements.

2. Mathematical description of the incineration processes

The incineration process of solid wastes can be divided into four successive sub-processes: evaporation of moisture from the solids, volatile release/char formation, burning of the hydrocarbon volatiles in the gaseous space, and the gasification of char particles. These four processes may overlap to some extent. Mathematical models for each of these processes are described as follows.

2.1. Moisture evaporation

Waste solids are first heated up by over-bed radiation when they enter the incinerator and the moisture is released as evaporation proceeds. Moisture can also be driven out by the mass exchange between the wet solids and the drier air-flow from the grate. The rate of moisture release from solids can be expressed as:

\[ R_{\text{evp}} = S_\text{a}h_\text{a}(C_{\text{w,s}} - C_{\text{w,g}}) \text{ when } T_s < 100 \, ^\circ\text{C} \]  

or

\[ R_{\text{evp}} = Q_{\text{cr}}/H_{\text{evp}} \text{ when } T_s < 100 \, ^\circ\text{C} \]

\[ Q_{\text{cr}} \] includes both convection and radiation heat transfer which is expressed as

\[ Q_{\text{cr}} = S_\text{a}(h'_\text{a}(T_g - T_s) + \epsilon_s \sigma_b(T_{\text{env}}^4 - T_s^4)) \]

2.2. Waste devolatilization

Volatile matter in municipal waste is normally much higher than that in coals. A typical figure of volatile content in municipal refuse is 85% of the combined volatile matter and fixed carbon, compared with 37% for an average industrial coal (Field and Rolfe, 1969). Depending on specific conditions, volatile yield could range from 10% to as high as 60% of the original mass, and the gases are composed mainly of hydrocarbon (CmHn), CO, CO2, H2, O2 and other trace compounds:

Waste → Volatile (CmHn, CO, H2, CO2, O2, etc) + Char

Established mathematical models have included the simple one-step global reaction mechanism, competing parallel two reaction models and the Gauss distribution activation energy model (Smoot and Pratt, 1979). Due to the diversity in the types of waste materials to be burned in municipal incinerators, the continuous Gaussian distribution of activation energies is considered more reasonable,
(\nu_\infty - \nu) \nu_\infty = \left[\sigma(2\pi)^{1/2}\right]^{-1} \int_0^\infty \exp\left[-\left(\int_0^t kdt\right) f(E) dE\right] \\
(3a)

\text{with}

f(E) = \left[\sigma(2\pi)^{1/2}\right]^{-1} \exp\left[-(E - E_0)/2\sigma^2\right] \\
(3b)

and

k = A \exp(-E/RT_s) \\
(3c)

2.3. Combustion of volatiles

Volatile products emerging from the particle surfaces have to first mix with the surrounding air before any combustion can take place. Gas combustion takes place in voids that could be regarded as long inter-connected channels whose widths are comparable to particle diameters in bed. Obviously the burning of the volatile hydrocarbon gases is limited not only by the reaction kinetics (temperature dependent) but also by the mixing-rate of the gaseous fuel with the under-fire air. The effect of the mixing process has been fully investigated elsewhere (Yang et al., in preparation), and the cell-averaged mixing rate was correlated as:

\[ R_{\text{mix}} = C_{\text{mix}} \rho_g \left\{ 150 \frac{D_G(1 - \phi)^{2/3}}{l_p \phi} + 1.75 \frac{V(1 - \phi)^{1/3}}{l_p \phi} \right\} \]

\[ \min \left\{ \frac{C_{\text{fuel}}}{S_{\text{fuel}}}, \frac{C_{\text{O}_2}}{S_{\text{O}_2}} \right\} \quad (4a) \]

within the bed, and

\[ R_{\text{mix}} = R_{\text{mix}}^{\text{free}} (2.8e^{-0.2y^{++}} - 18e^{-2y^{++}}) \min \left\{ \frac{C_{\text{fuel}}}{S_{\text{fuel}}}, \frac{C_{\text{O}_2}}{S_{\text{O}_2}} \right\} \quad (4b) \]

\[ y^{++} = y^{+}/d_{ch} \quad (4c) \]

in the freeboard area immediately next to the bed surface, where \( R_{\text{mix}}^{\text{free}} \) is calculated from Eq. (4a) without the species concentration terms at the bed surface.

Furthermore, devolatilization yields a wide range of different gaseous fuels, which for reasons of complexity cannot be treated separately. Therefore a simplified approach is adopted in which carbon monoxide and a representative hydrocarbon are assumed to be the only combustible products from devolatilization. So the gas phase reactions are reduced to

\[ \text{CO} + 1/2\text{O}_2 \rightarrow \text{CO}_2 \quad (5) \]

and

\[ \text{C}_m\text{H}_n + (m/2 + n/4) \text{O}_2 \rightarrow m\text{CO} + n/2 \text{H}_2\text{O} \quad (6) \]

The temperature-dependent burning rates or kinetic rates, \( R_{co} \) and \( R_{\text{cmHn}} \), are given by Howard et al. (1973) for CO and Siminski et al. (1972) for \( C_m\text{H}_n \) as

\[ R_{co} = 1.3 \times 10^{14} C_{\text{CO}} C_{\text{H}_2\text{O}}^{0.5} C_{\text{O}_2}^{0.5} \]

\[ (7) \]

and

\[ R_{\text{cmHn}} = 59.8 T_0 \rho^{0.3} C_{\text{cmHn}}^{0.5} C_{\text{O}_2} \exp(-12200/T_0) \]

\[ (8) \]

The actual reaction rates of volatile species are calculated as the minimum of the temperature-dependent kinetic rates and their mixing-rates with oxygen:

\[ R = \text{Min}[R_{\text{kinetic}}, \ R_{\text{mix}}] \]

\[ (9) \]

2.4. Gasification of char

Char forms as volatiles escape from the particles. The primary products of char gasification are CO and CO2,

\[ \text{C(s)} + \alpha \text{O}_2 \rightarrow 2(1 - \alpha) \text{CO} + (2\alpha - 1) \text{CO}_2 \]

\[ (10) \]

with the ratio of CO and CO2 as (Arthur, 1951):

\[ \text{CO/CO}_2 = 2500 \exp(-6420/T) \]

\[ (11) \]

for temperatures between 730 and 1170 K. Ratios outside of this temperature range are bracketed to the limits.

The char consumption rate is expressed as (Smoot and Pratt, 1979):

\[ R_{\text{C(s)}} = C_{\text{CO}_2} /(1/k_c + 1/k_d) \]

\[ (12) \]

3. Transport equations for gas and solid phases

3.1. Gas-phase: turbulent fluid flow and species transport in a packed bed of solids

Turbulent flow in a packed bed of solids is quite different from flow inside a duct. Large-scale turbulence is impossible within a packed bed due to the restriction imposed by the narrow and irregular channels through which fluid has to pass, and the spectrum of turbulence moves towards smaller scale, which means an increase in dissipation rate of turbulent kinetic energy. Particle resistance to the flow becomes much more important and the subsequent large pressure drop will dominate the flow field distribution.

Peters (1995) has summarised the general governing equations for both gas and solid phases in a moving packed bed. Readers can refer to elsewhere (Peters, 1995) for more detailed description. The hydraulic radius theory of porous media applies here and initial
work was carried out by Darcy, who related flow in porous media to the governing pressure gradient through viscosity and porosity as an essential parameter of porous media, and the continuity and momentum equations of the gas-phase in a packed bed can be written as:

**Continuity:**

\[
\frac{\partial \rho_g}{\partial t} \nabla \cdot (\rho_g (V_g - V_B)) = S_g \quad (13)
\]

**Momentum:**

\[
\frac{\partial \rho_g}{\partial t} \nabla \cdot \rho_g (V_g - V_B) V_g = -\nabla p_g F(v) \quad (14)
\]

The function \( F(v) \) accounts for different flow regimes and was expressed as,

\[
F(V_g) = \begin{cases} 
-\mu V_g & \text{if } \text{Re} < (\text{Darcy}) \\
-\frac{\mu}{K} V_g - \rho_g CV_g |V_g| & \text{if } \text{Re} \geq (\text{Forchheimer}) 
\end{cases} \quad (15)
\]

where \( \kappa \) is the permeability and \( K \) and \( C \) are constants of the form (Hunt and Tien, 1988; Vafai and Sozen, 1990):

\[
K = \frac{d_p^3 \phi^3}{150(1 - \phi)^2} \quad (16)
\]

\[
C = 1.75(1 - \phi) \frac{d_p \phi^3}{d_p \phi^3} \quad (17)
\]

Similarly, the species transport equation can be written as:

\[
\frac{\partial \rho_{gb}}{\partial t} Y_{bg} + \nabla \cdot (\rho_g (V_g - V_B) Y_{bg}) = \bullet (D_{gb} \cdot (\rho_g Y_{bg})) + S_{gb} \quad (18)
\]

The fluid dispersion coefficient is considered to consist of diffusional and turbulent contributions. The diffusion contribution corresponds to the effective diffusion coefficient at zero flow and is isotropic. In the range of laminar flow, \( \text{Re} < 1 \), the dispersion coefficient only consists of a diffusional contribution. At Reynolds number greater than 5, flow becomes turbulent. Hence the cross-flow and in-flow dispersion coefficients are given by the following equations (Wakao and Kaguei, 1982):

\[
D_t = D_{ax} = E^0 \text{ for } \text{Re} < 1 \quad (19a)
\]

\[
D_t = E^0 + 0.1d_p V_g \text{ for } \text{Re} > 5 \quad (19b)
\]

\[
D_{ax} = E^0 + 0.5d_p V_g \text{ for } \text{Re} > 5 \quad (19c)
\]

### 3.2. Gas-phase: transport equation for heat transfer

The energy equation for the gas phase in a packed bed can be written as:

\[
\frac{\partial \rho_g M_2}{\partial t} + \nabla \cdot (\rho_g (V_g - V_B) H_g) = \nabla \cdot \left( \lambda_g \nabla T_g \right) + \frac{D_p}{D_t} - F(V_g) V_g + Q_n \quad (20)
\]

The thermal dispersion coefficient \( \lambda_g \) consists of diffusional and turbulent contributions in a similar way to species dispersion, and can be expressed as (Wakao and Kaguei, 1982):

\[
\begin{align}
\lambda_{rg} & = \lambda^0 \text{ for laminar flow } (\text{Re} < 1) \\
\lambda_{rg} & = \lambda^0 + 0.1d_p V_g \rho_g C_{pg} \text{ for turbulent flow } (\text{Re} < 5) \quad (21b)
\end{align}
\]

\[
\begin{align}
\lambda_{axg} & = \lambda^0 \text{ for laminar flow } (\text{Re} < 1) \\
\lambda_{axg} & = \lambda^0 + 0.5d_p V_g \rho_g C_{pg} \text{ for turbulent flow } (\text{Re} < 5) \quad (22b)
\end{align}
\]

### 3.3. Solid-phase: particle movement and heat transfer in a packed bed

The equations of particle movement in a packed bed take similar forms as those for a fluid and are expressed as follows:

**Continuity:**

\[
\frac{\partial \rho_{sb}}{\partial t} + \nabla \cdot (\rho_{sb} (V_s - V_B)) = S_s \quad (23)
\]

**Momentum:**

\[
\frac{\partial \rho_{sb} V_s}{\partial t} + \nabla \cdot (\rho_{sb} (V_s - V_B) V_s) = -\nabla \cdot \sigma - \nabla \cdot \tau + \rho_{sb} F + A \quad (24)
\]

The fourth term on the right hand side of Eq. (24) is included to account for particle random movements (mixing) caused by the mechanical disturbance of the moving grate and other random sources. At the present stage, Eq. (24) is not being solved for the particle velocities in the bed due to lack of proper models for the stress terms. Instead the horizontal movement of the bed is predefined and the vertical particle velocity in the bed is then obtained using Eq. (23). The conservation equation for the solid-phase species is expressed as:
\[ \frac{\partial \rho_s Y_{is}}{\partial t} + \nabla \cdot (\rho_s (V_s - V_b) Y_{is}) = \nabla \cdot (D_s \nabla (\rho_s Y_{is})) + S_{ys} \]  
\[ \text{(25)} \]

The energy equation for solid-phase is:
\[ \frac{\partial \rho_s H_{is}}{\partial t} + \nabla \cdot (\rho_s (V_s - V_b) H_{is}) = \nabla \cdot (\lambda_s \nabla T_s) + \nabla \cdot q_{sh} Q_{sh} \]  
\[ \text{(26)} \]

The source term \( Q_{sh} \) summarises effects such as heat transfer between gas and particles and heat generation due to heterogeneous combustion.

The solid bed effective thermal conductivity \( \lambda_s \) in Eq. (26) consists of two parts, the solid material conductivity \( \lambda_{sb} \), and thermal transport caused by particle random movements, \( \lambda_{sm} \).

\[ \lambda_s = \lambda_{sb} + \lambda_{sm} = (1 - \phi) \dot{\lambda}_i + \dot{\lambda}_{sm} \]  
\[ \text{(27)} \]

The transport coefficients of the solid-phase, i.e. \( \mu_s \), \( D_s \) and \( \dot{\lambda}_{sm} \) in a packed bed have to be estimated. We can define “Particle Prandtl Number”, \( Pr_s \), and “Particle Schmidt Number”, \( Sc_s \), by analogy with the fluid phase, and by assuming:

\[ Pr_s = Sc_s = 1 \]  
\[ \text{(28)} \]

We get

\[ \mu_s = \rho_s D_s \] and \( \dot{\lambda}_{sm} = \mu_s C_{ps} = \rho_s C_{ps} D_s \]  
\[ \text{(29)} \]

\( D_s \) can be approximated by lab measurements and is affected by physical properties of waste, grate type and operation conditions of furnaces.

3.4. Radiation heat transfer in the bed

Radiation is the major mechanism of heat transfer between solid particles in a packed bed, and a proper model has to be developed to simulate the process. The already widely used flux model (Smoot and Pratt, 1979) for gaseous and entrained-flow combustion seems to be the first choice, although development of more appropriate models is needed in the future. A four-flux radiation model is presented in the following:

\[ \frac{dI_y^+}{dz} = -(k_a + k_s)I_y^+ + \frac{1}{4} k_s E_b + \frac{1}{4} k_s \left( I_y^+ + I_y^- + I_z^+ + I_z^- \right) \]  
\[ \text{(30a)} \]

\[ \frac{dI_y^-}{dz} = -(k_a + k_s)I_y^- + \frac{1}{4} k_s E_b + \frac{1}{4} k_s \left( I_y^+ + I_y^- + I_z^+ + I_z^- \right) \]  
\[ \text{(30b)} \]

\[ \frac{dI_z^+}{dz} = -(k_a + k_s)I_z^+ + \frac{1}{4} k_s E_b + \frac{1}{4} k_s \left( I_y^+ + I_y^- + I_z^+ + I_z^- \right) \]  
\[ \text{(30c)} \]

\[ \frac{dI_z^-}{dz} = -(k_a + k_s)I_z^- + \frac{1}{4} k_s E_b + \frac{1}{4} k_s \left( I_y^+ + I_y^- + I_z^+ + I_z^- \right) \]  
\[ \text{(30d)} \]

\( k_s \) is assumed zero as the first approximation, and \( k_a \) is taken as

\[ k_a = -\frac{1}{D_p} \ln(P) \]  
\[ \text{(31)} \]

4. Solving

Except for radiation, the governing equations described above are generalised into a standard form:

\[ \frac{\partial \Phi}{\partial t} + \nabla \cdot (\rho V \Phi) = \nabla \cdot (\lambda \nabla \Phi) + S_{\Phi} \]  
\[ \text{(32)} \]

The whole geometrical domain of the bed is divided into a number of small cells and Eq. (32) discretised over each cell and solved using the SIMPLE algorithm (Patankar, 1980). For each cell the equation becomes:

\[ a_{ij} \Phi_{ij} + a_{i-1,j} \Phi_{i+1,j} + a_{i,j-1} \Phi_{i,j-1} + a_{i,j+1} \Phi_{i,j+1} \]

\[ = S_{ij} \] (i = 1, M; j = 1, N)  
\[ \text{(33)} \]

The radiation equations are solved by the fourth-order Runge-Kutta method (Harnbeck, 1975). A graphically interactive computer program, The Fluid Dynamic Incinerator Code (FLIC) has been developed to carry out the calculations.

5. Experimental

Fig. 1 shows the schematic diagram of the experimental rig which was constructed in order to study the combustion of municipal solid waste. The combustion rig consists of a cylindrical combustion chamber holding about 2 kg of sample waste. The height of the chamber is 1.5 m with an internal diameter of 20 cm. The chamber is mounted on a weighing balance to permit the burning rate to be determined. The internal lining of the chamber is made of Inconel 600 nickel alloy. Inconel 600 was chosen because of its resistance to corrosion at high temperature and its ability to withstand the high temperature of the combustion, which is expected to reach 1200 °C. The outside casing is a stainless steel cylinder...
of 420 mm outside diameter and 30 mm thick wall. Sandwiched between the Inconel lining and the casing is a thick layer of insulation (Kaowool thermal ceramic blanket) which can withstand high temperature up to 1260 °C. Underfire air preheated to 100 °C was supplied through a stainless steel perforated plate with 7% open area and secondary air was injected at three different positions in the top section of the chamber. The waste sample was loaded on the grate to a depth of 580 mm from the grate. A gas burner is placed at 45° towards the waste at 750 mm above the grate and is used to initiate the burning process and maintain a freeboard temperature in the chamber at about 800 °C. Gas samples can also be drawn from any sampling positions for continuous measurement during the experiment. The gas was analysed for O₂, CO, CO₂, NOₓ and volatile organic carbon (VOC). Thermocouples are located at fixed positions within the fuel bed and temperatures were recorded continuously.

The simulated waste was a mixture of wood, cardboard, paper and vegetable matter. Its analysis is presented in Tables 1 and 2. Air flow rate into the bottom of the bed is 0.052 kg/(m² s) at 373 K.

### Table 1
**Proximate analysis (as fired; %weight)**

<table>
<thead>
<tr>
<th>Moisture</th>
<th>Volatile matter</th>
<th>Fixed carbon</th>
<th>Ash</th>
<th>Average size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>48.5</td>
<td>41.53</td>
<td>7.07</td>
<td>2.9</td>
<td>12.0</td>
</tr>
</tbody>
</table>

### Table 2
**Ultimate Analysis (daf; %weight)**

<table>
<thead>
<tr>
<th>Carbon</th>
<th>Hydrogen</th>
<th>Oxygen</th>
<th>Nitrogen</th>
<th>Sulphur</th>
</tr>
</thead>
<tbody>
<tr>
<td>46.63</td>
<td>5.77</td>
<td>47.06</td>
<td>0.27</td>
<td>0.27</td>
</tr>
</tbody>
</table>

6. Results and discussion

For mathematical modeling the calorific value of the waste is calculated as 6873 kJ/kg from elemental composition. Bed bulk density is measured as 130 kg/m³. Modified devolatilization rate parameters were adopted as (Anthony et al., 1975) \( A = 3.2 \times 10^{-9} \text{s}^{-1} \), \( E_0 = 203 \text{ kJ/mol} \) and \( \sigma = 39.26 \text{ kJ/mol} \). Rate parameters for char combustion are taken as \( A = 860 \text{ kg m}^{-2} \text{s}^{-1} \text{kPa}^{-1} \) and \( E/R = 18,000 \text{ K} \). \( C_{\text{mix}} \) in Eq. (4a) is optimized as 0.5. The whole bed volume is divided into 25×50 cells.

Fig. 2 shows the predicted solid temperature distribution in the stationary bed. The vertical co-ordinate represents the position in mm above the grate and the horizontal co-ordinate represents the time of duration in seconds. This rule applies also to Figs. 3 and 4. For most of the time duration, changes in bed temperature are only seen in a thin layer about 60–100 mm in thickness along the bed height, i.e. 5–8 times of the average particle diameter. The most severe temperature change occurs at half of the layer thickness indicating the position of the flame front. The highest solid temperature is around 1150 K and located between 2250 and 2800 mm along the bed length, where moisture in the waste has been driven out completely and fixed carbon begins to burn vigorously. The bed surface here is the hottest because there is only a very small amount of ash in the waste. In this region it can be calculated that the burning front propagates downwards from the bed surface at a speed about 15 mm/min.

Fig. 3 shows the predicted rate of moisture evaporation in the stationary bed against time. Because of the high moisture content of the waste (48% by mass), the moisture evaporation process becomes a very significant factor in this waste incineration, and the rate of drying out largely controls the overall mass loss in the bed. The moisture content in the waste begins to reduce at the bottom of the bed as hot air (373 K) from the grate drives it out, but the bulk of moisture evaporation is at the upper part of the bed where intensive heat from combustion and over-bed media radiation raise the solid temperature quickly. Thus the figure shows that intensive evaporation occurs within a layer about 70 mm in depth and not far away from the bed surface. The moisture evaporation process extends about two-thirds of the whole time duration.

Fig. 4 demonstrates the predicted release rate of volatile material from solid particles in the packed bed. Devolatilization begins shortly after the waste feed enters the furnace. A combination of high temperature and freshness of the waste material produces a high release rate per unit volume at this early stage of particle devolatilization. Further on, the devolatilization rate reduces to a more or less stable level until around 2200 s of the time duration. At this point the moisture in the waste has been completely driven out, bed temperature...
Fig. 2. Predicted solid temperature distribution vs. time duration in the bed.

Fig. 3. Predicted moisture evaporation rate vs. time duration in the bed.

Fig. 4. Predicted release rate of volatile gases vs. time duration in the bed.
rises quickly and a higher release rate of the volatile material is predicted as a result. Remembering that the volatile material comprises over 40% of the total waste weight, i.e. 85% of the combustible mass, it is not surprising that the devolatilization process extends over nearly the whole combustion time. Nevertheless, its intensive part is confined to a thin layer about 40–60 mm in depth at the top of the bed, which is 4–5 times the average particle size.

The waste mass loss against time is shown in Fig. 5. A roughly linear relationship between mass loss and time is observed from time zero till $t = 2000$ s, corresponding to the point where there is no moisture left in the waste material. This indicates that the mass loss rate is roughly constant during this time period, which features mainly the moisture evaporation and release of volatiles from the solids. After this point, however, the mass loss accelerates dramatically because all the heat absorbed by solids is now used to raise the solid temperature only and hence the high rate of particle devolatilization.

Furthermore, the mass loss rate gradually levels off as the devolatilization process approaches completion and the residual fixed carbon begins to burn. In this specific case the combustion of fixed carbon or char comprises only a small portion of the whole waste burning process due to the relatively small amount (7% by mass). The whole combustion process is complete when $t = 3300$ s.

Temperature profiles for the whole computation region are shown in Fig. 6 at four different heights above the grate. Lines of different pattern represent the predicted profiles at normal bed conditions without channeling, and the symbols represent measured profiles. According to measurement, temperature at 30 mm above the initial bed surface (i.e. 610 mm above the grate) rose very quickly during the first 320 s, to 1023 K, and this time duration can be considered as the ignition time for the waste material. A gradual increase in temperature followed until $t = 850$ s where roughly constant temperature (about 1230 K) was reached. Temperature at 600 mm began to fall slowly at $t = 2100$ s when the bed surface dropped far below the measuring point and radiation heat loss to the cooler walls of the furnace significantly reduced the gas temperature. A similar pattern was observed for the temperature at 520 mm above the grate, and the highest temperature measured was the same as at 620 mm above the grate. The only difference is that the process was delayed for a certain time, about 350 s, compared to the latter, an expected phenomenon under ideal circumstances. The situation at 430 mm above the grate was no longer following such ideal pattern, however. A longer delay of the combustion process was obvious at the measuring point, about 700 s compared to the previous point (520 mm), and also, the highest temperature was much lower than that at the two measuring points above, by 170 K. This irregularity of the combustion process is considered to be caused by channeling in the bed. Channeling in the bed causes the feed air to by-pass the normal void portions between the bed particles and escape through short-cuts or big holes in the bed to the over-bed space without the expected reaction with those particles. Because of this and depending on the scale of channeling, the temperature distribution across bed is no longer uniform, and the bed conditions drift away from the ideal uniformity.

The measuring point nearest to the grate was located 90 mm above the grate. A significant temperature increase started from $t = 2200$ s, and the temperature reached its highest, 1327 K, around $t = 2800$ s. This is higher than the maximum temperature measured at previous points (about 1230 K). Model prediction indicates that the combustion mode in this region was basically fixed-carbon or char burning. And because of the small percentage of fixed-carbon in the feed waste (7% by mass), the char quickly burnt out, and the bed temperature decreased sharply afterwards.

Figs. 7–9 demonstrate dry and molar percentages of CO$_2$, O$_2$ and CO, respectively, against time at position 430 mm above the grate. The patterns of CO$_2$ and O$_2$ varied according to measurements. Starting at $t = 950$ s where the flame front was just passing, oxygen level dropped dramatically and carbon dioxide rose sharply within the following 300 s. Estimation shows that this time duration corresponds to 60 mm downward bed movement along the height, or equivalent bed height of the same amount above the flame front when it is passing the 430 mm point. The numerical prediction (see
Figs. 2–4) clearly indicates a reacting layer of nearly the same height, and very good agreement between calculation and measurement is reached in this respect.

The fluctuations in CO\textsubscript{2} and O\textsubscript{2} concentrations above the bed surface are clearly seen in Figs. 7 and 8 from measurement. The biggest fluctuation was at \( t = 2200 \text{ s} \) where the oxygen level fell to a 3% minimum at the highest CO\textsubscript{2} of 12% and then rose to 15% maximum with lowest CO\textsubscript{2} at 4.5%. This specific bed characteristic of fluctuations in gaseous concentrations was due to the channeling phenomenon in the bed and a subsequent catastrophic change (e.g. sudden local collapse...
of the bed) in local bed conditions including void fraction, particle surface area, etc. Bed channeling is an unwanted phenomenon because it reduces the efficient contact between feed air and burning solids, resulting in low combustion efficiency. Yet this is a very important phenomenon in practical situations although many designers of MSW incinerators try to avoid this by proper grate design that creates right disturbances in the bed.

The molar fraction of carbon monoxide against time at 430 mm above the grate is shown in Fig. 9. After passage of the flame front and above the bed surface, the measured CO level was between 1000 and 3000 ppmv before \( t = 2200 \) s. However, the CO level was well above 3000 ppmv (which is the upper limit of the instrument detection) after \( t = 2200 \) s, and according numerical predictions, could reach as high as 6% by volume. This high CO region corresponds the end of moisture evaporation in the waste (see Fig. 3), faster devolatilization of the solids (see Fig. 4) and fierce combustion of volatile gases and fixed carbon. The insufficient air supply from the grate is the reason for the high CO concentration in a region which is towards the end the whole combustion process.

7. Conclusions

Governing equations for mass, momentum and heat transfer in a moving packed bed are described and mathematical descriptions for the processes of moisture evaporation, particle devolatilisation, volatile combustion and char burnout are presented. Radiation in the bed is simulated by a four-flux model. The burning rates of volatile hydrocarbons in a packed bed are limited by not only the reaction kinetics but also the mixing of the volatile fuel with the feed air. The cell-averaged mixing rate is correlated to a number of parameters including local void fraction, gas velocity and a length scale comparable to particle size or the size of void space. A correlation equation is also included to calculate the mixing in the freeboard area immediately next to the bed surface. A bench-top stationary waste incinerator was built and test runs made in which total mass loss from the bed, temperature and gas composition at different locations along the bed height were measured. A 2-D bed-modelling program, FLIC, was developed which incorporates the various sub-process models and solves the governing equations for both gases and solids.

The total mass loss from the bed was dominated by moisture evaporation and volatile release from the waste solids. A roughly linear relationship was found between the total mass loss and the time period until moisture was totally driven out of the waste. The majority of the physical and chemical processes were confined within a layer of 60–100 mm along bed height at the top surface, which is 5–9 times the average particle size in the bed. This combustion zone was around 60 mm in depth at the bed surface. The maximum bed temperature was around 1200 K. The CO level in the bed was between 1000 and 3000 ppmv for the first two-thirds of the total combustion time, and rose highly shortly after due to the increased rate of devolatilisation and fierce fixed-carbon burnout. \( \text{CO}_2 \) and \( \text{O}_2 \) fluctuated at the measuring point, and big differences were observed from the minimum to the maximum due to channelling and subsequent ‘catastrophic’ changes in the local bed conditions.

Reasonably good agreement between modelling and measurements has been achieved. Yet the modelling work is complicated by the channelling phenomenon in the bed that inhibits mixing between combustible gases and air, and causes a lower combustion efficiency of hydrocarbons and increased CH emission in the flue gas. Numerical simulations without consideration of the channelling effect produced very good agreement with experiments concerning the total mass loss, but significant discrepancy exists for temperature and gas composition profiles.

Further efforts are needed to incorporate aspects of ‘catastrophe’ theory into the current model to predict the fluctuating character of gas composition caused by channelling and sudden changes in bed status.

8. Notation

\( A \) momentum exchange due to particle random movement, N/m\(^3\)
\( C \) constant; molar fractions of species (fuel, oxygen)
\( C_{pg} \) specific heat capacity of the gas mixture, J/kg K
\( C_{mix} \) mixing-rate constant
\( C_{w,g} \) moisture concentration in the gas phase, kg/m\(^3\)
\( C_{w,s} \) moisture concentration at the solid surface, kg/m\(^3\)
\( D_{ax} \) cross-flow dispersion coefficient in bed, m\(^2\)/s
\( \bar{d}_{ch} \) channel size, m
\( D_g \) molecular diffusion coefficient of volatile hydrocarbons in air, m\(^2\)/s
\( D_{tg} \) dispersion coefficients of the species \( Y_{tg} \), m\(^2\)/s
\( d_p \) particle diameter, m
\( D_r \) in-flow dispersion coefficient in bed, m\(^2\)/s
\( D_s \) particle mixing coefficient due to random movements of particles in the bed, m\(^2\)/s
\( E \) activation energy, kJ/mol
\( E_0 \) the central value of activation energy in a Gaussian distribution, kJ/mol
\( E^0 \) effective diffusion coefficient, m\(^2\)/s
\( E_b \) black-body radiation, W/m\(^2\)
\( g \) the gravity
\( H_{exp} \) evaporation heat of the solid material, J/kg
\( H_e \) gas enthalpy, J/kg
\( H_s \) solid-phase enthalpy, J/kg
$h_s$ convective mass transfer coefficient between solid and gas, m/s

$h'_s$ convective heat transfer coefficient between solid and gas, W/m$^2$ s

$I_x^*, I_x^*, I_y^*$, and $I$ radiation intensities in the four directions, W/m$^2$

$k_a$ absorption coefficient, 1/m

$k_s$ scattering coefficients respectively, 1/m

$k_d$ rate constants due to diffusion, kg/m$^2$s

$k_c$ rate constants due to chemical kinetics, kg/m$^2$s

$l_p$ turbulence length scale in the bed, m

$\rho_g$ gas pressure, Pa

$P$ pressure, atm

$Q_{cr}$ heat absorbed by the solids, W

$Q_h$ heat loss/gain of the gases, W/m$^3$

$Q_{sh}$ thermal source term for solid phase, W/m$^3$

$q_r$ radiative heat flux, W/m$^2$

$R_{evp}$ moisture evaporation rate, kg/s

$R_{mix}$ overall mixing-rate of gaseous phase in the bed, kg/m$^3$s

$S$ stoichiometric coefficients in reactions

$S_v$ particle surface area, m$^2$

$S_g$ mass source term, kg/m$^3$s

$S_{y_{ig}}$ mass sources due to evaporation, devolatilization and combustion, kg/m$^3$s

$S_{y_is}$ source term, kg/m$^3$s

$S_p$ general source term

$T_{env}$ environmental temperature, K

$T_g$ gas temperature, K

$T_s$ solid temperature, K

$V$ superficial gas velocity, m/s

$V_B$ velocity of a moving boundary, m/s

$V_g$ superficial gas velocity (vector), m/s

$V_s$ particle velocity, m/s

$Y_{is}$ mass fractions for gaseous species (e.g. H$_2$, H$_2$O, CO, CO$_2$, C$_n$H$_m$, ...)

$Y_{ts}$ mass fractions of particle compositions (moisture, volatile, fixed carbon and ash)

$y^+$ distance above the bed-top surface, m

8.1. Greek symbols

$\epsilon_s$ system emissivity

$\sigma$ deviate of activation energy in a Gaussian distribution, kJ/mol

$\sigma_b$ Boltzmann radiation constant, W/m$^2$K$^4$

$\sigma$ normal stress exerted on the particles, N/m$^2$

$\nu$ the remaining volatile amount at time $t$

$\nu_{\infty}$ the ultimate yield of volatile

$\delta$ void fraction in the bed

$\phi$ parameter to be solved

$\rho$ density, kg/m$^3$

$\rho_g$ gas density, kg/m$^3$

$\rho_{sb}$ solid bulk density in the bed, kg/m$^3$

$\lambda$ general diffusion coefficient

$\lambda_g$ thermal dispersion coefficient, W/mK

$\kappa_s$ effective thermal conductivity of the solid bed, W/mK

$\lambda^0$ effective thermal diffusion coefficient, W/mK

$\tau$ shear stress exerted on the particles, N/m$^2$

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References


