ABSTRACT

This paper considers the design and optimization of thermal systems on the basis of the mathematical and numerical modeling of the system. Many complexities are often encountered in practical thermal processes and systems, making the modeling challenging and involved. These include property variations, complicated regions, combined transport mechanisms, chemical reactions, and intricate boundary conditions. The paper briefly presents approaches that may be used to accurately simulate these systems. Validation of the numerical model is a particularly critical aspect and is discussed. It is important to couple the modeling with the system performance, design, control and optimization. This aspect, which has often been ignored in the literature, is considered in this paper. Design of thermal systems based on concurrent simulation and experimentation is also discussed in terms of dynamic data-driven optimization methods. Optimization of the system and of the operating conditions is needed to minimize costs and improve product quality and system performance. Different optimization strategies that are currently used for thermal systems are outlined, focusing on new and emerging strategies. Of particular interest is multi-objective optimization, since most thermal systems involve several important objective functions, such as heat transfer rate and pressure in electronic cooling systems. A few practical thermal systems are considered in greater detail to illustrate these approaches and to present typical simulation, design and optimization results.
INTRODUCTION

An area which has received relatively little attention is that of design and optimization of thermal systems, even though extensive work has been done on the simulation of thermal processes. However, the mathematical and numerical modeling of the complex transport phenomena that arise in typical practical thermal systems are complicated due to the material characteristics, complicated multiple domains, intricate boundary conditions and additional mechanisms such as surface tension, chemical reactions, and phase change. The numerical simulation of such processes is discussed in this paper, along with system design and optimization based on the results obtained. Typical results on simulation, design and optimization are presented and discussed for a few important practical processes, such as optical fiber manufacturing, chemical vapor deposition for the fabrication of thin films, and the cooling of electronic systems. Validation of the model is an important aspect and is discussed, considering comparisons with analytical, numerical and experimental results. An accurate and valid numerical simulation of the underlying processes is needed for improving existing systems and for the design and optimization of future systems.

Many practical systems are based on a consideration of heat transfer, thermodynamics, fluid mechanics and mass transfer to a significant extent in their analysis and characterization and are thus termed thermal systems. These systems are important in many different applications, such as manufacturing, energy, cooling of electronic equipment, transportation, air conditioning, refrigeration and heat transfer equipment. Three important thermal systems are sketched in Figs. 1-3. These are the optical fiber manufacturing system, which involves drawing, cooling and coating of the fiber, drawn from a specially fabricated glass cylinder, chemical vapor deposition of materials like Silicon for the fabrication of thin films, and a simple system indicating electronic components in a channel with a vortex promoter to enhance cooling. The analysis of such thermal systems is often complicated because of the complex, nonlinear, nature of fluid flow and of heat and mass transfer that govern these systems. Approximations and idealizations are often used to simplify these equations and thus model the system for numerical simulation. It is important to validate the models to ensure their accuracy and to use the results generated in the design and optimization of these systems, as discussed by Stoecker (1989), Bejan et al. (1996) and Jaluria (1998).

MODELING AND SIMULATION

The underlying mechanisms for thermal processes are governed by the conservation laws for mass, momentum and energy. A radiative source term arises for non-opaque materials like glass and water, which emit and absorb energy as function of the wavelength. Also, viscous dissipation effects are important for flow of highly viscous materials like glass and polymers used in fiber coating due to the large viscosity of the material. For instance, the governing equations for axisymmetric conditions, developed in cylindrical coordinates for both the glass and the inert gas, for optical fiber drawing are given by Cheng and Jaluria (2005) as:

Fig. 1. Sketch of an optical fiber drawing system.

Fig. 2. An impingement type chemical vapor deposition (CVD) system.

Fig. 3. Multiple heat sources in a channel with a vortex promoter, approximating an electronic system.
problems, the primitive variables are used more efficiently (Minkowycz and Sparrow, 1997; Jaluria and Torrance, 2003).

The vorticity equation, the stream function, vorticity and energy equations can then be solved using a non-

The transport processes in a wide range of basic and applied problems have been investigated, see, for instance, Poulikakos (1996) and Jaluria (2003) for materials processing systems. A few typical results are

\[ \frac{\partial v}{\partial z} + \frac{1}{r} \frac{\partial (ru)}{\partial r} = 0 \]  

\[ \frac{\partial v}{\partial t} + \frac{u}{r} \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial z} = \frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left[ rv \left( \frac{\partial v}{\partial r} + \frac{\partial u}{\partial z} \right) \right] + 2 \frac{\partial}{\partial z} \left[ v \frac{\partial v}{\partial z} \right] \]  

\[ \frac{\partial u}{\partial t} + \frac{u}{r} \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{2}{r} \frac{\partial}{\partial r} \left( rv \frac{\partial u}{\partial r} \right) + \frac{\partial}{\partial z} \left[ v \frac{\partial u}{\partial z} \right] - \frac{2 \nu u}{r^2} \]  

\[ \rho C_p \left( \frac{\partial T}{\partial t} + \frac{u}{r} \frac{\partial T}{\partial r} + \frac{v}{r} \frac{\partial T}{\partial z} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( rK \frac{\partial T}{\partial z} \right) + \frac{\partial}{\partial z} \left( K \frac{\partial T}{\partial z} \right) + \Phi + S_r \]  

where the viscous dissipation term, \( \Phi \), is given by

\[ \Phi = \mu \left[ 2 \left( \frac{\partial u}{\partial r} \right)^2 + u \frac{\partial v}{\partial z} + \left( \frac{\partial u}{\partial z} \right)^2 \right] \]  

This equation shows a strong, exponential, variation of \( v \) with temperature. Variations in all the other relevant properties of glass need to be considered as well. Numerical models that can simulate such strong variations in the properties are needed for an accurate simulation of the process. The radiative source term for the glass is often approximated in terms of bands with constant absorption over each band (Chen and Jaluria, 2007).

Material properties, though critical to the accuracy of any numerical simulation, are often not available to desired accuracy. Frequently, the data are available at conditions that may be different from those for the actual process, thus limiting the accuracy of the simulation. As shown in Fig. 1, optical fibers are fabricated by heating a specially designed silica glass preform in a cylindrical furnace above the softening point \( T_{\text{melt}} \) of around 1900K and applying an axial draw tension so that the preform is drawn into a fiber through a neck-down region (Paek, 1999). The material properties are strong functions of the temperature \( T \), the variation in the viscosity being the most critical one for the flow. An equation based on the curve fit of available data for kinematic viscosity \( \nu \) can be written for silica, in S.I. units, as

\[ \nu = 4545.45 \exp \left[ 32 \left( \frac{T_{\text{melt}}}{T} - 1 \right) \right] \]  

Similarly, expressions for other coordinate systems may be obtained. Energy input can also be provided by chemical reactions, as is the case in combustion, chemical vapor deposition (Mahajan, 1996) and thermal processing of reactive polymers. Then, the equations for the chemical species have to be solved and an accurate simulation of the chemical kinetics is critical to a satisfactory and validated modeling of the process.

SIMULATION RESULTS

The transport processes in a wide range of basic and applied problems have been investigated, see, for example, Poulikakos (1996) and Jaluria (2003) for materials processing systems. A few typical results are
presented here. Figure 5 shows typical computed results in terms of the velocity field in the neck-down region for three furnace lengths. The glass flow is seen to be smooth and well layered because of the high viscosity of the fluid. Typical temperature differences of order 100 °C arise across the preform for a diameter of around 5 cm. However, even these small differences can affect fiber quality because of the large, exponential, variation of viscosity with the temperature. The temperatures affect the neck-down shape and the generation of temperature-induced defects. Viscous dissipation is relatively small, but is mainly concentrated near the end of the neck-down and must be included in the model. Numerically calculated neck-down profiles, thermally induced defects and tension in the fiber have been obtained for single and multiple-layer optical fibers, as discussed by Paek (1999) and others mentioned earlier.

Another practical problem where simulation is important is the transport in a channel due to isolated, protruding, heat sources, as shown in Fig. 3. This configuration represents an electronic system, with the heat sources representing electronic components, such as chip assemblies. The geometry can be complicated due to the positioning and dimensions of the sources and of the promoter. Figure 6 shows the calculated flow and temperature distributions in such a channel. Interest lies in obtaining resonance between the frequencies of vortices generated by the promoter and by the sources. Even without resonance, a significant increase in heat transfer was obtained by Wang and Jaluria (2002). This system can be optimized in terms of operating conditions, like flow rate and heat input, and the design parameters, like source locations, vortex promoter geometry and materials used.

Similarly, the uniformity, and rate of deposition in a CVD reactor are dependent on the heat and mass transfer, and on the chemical reactions that are themselves strongly dependent on the temperature and concentration levels. These aspects have been investigated by several researchers, such as Eversteyn et al. (1970). Some typical results obtained for silicon deposition are shown in Fig. 7, indicating the temperature and flow fields for typical operating conditions. The resulting deposition was also computed by Yoo and Jaluria (2002) and compared with the experiments of Eversteyn et al. (1970), indicating fairly good agreement.

An important aspect in modeling and simulation is that of validation. This is particularly crucial in thermal systems because of simplifications and idealizations that are usually employed, lack of accurate material property data, and various complexities in the process. Unless the models are satisfactorily validated and the accuracy of the results obtained established, the models cannot be used as a basis for design and optimization. The physical behavior of the system, elimination of the effect of parameters like grid and time step, comparisons with available analytical or numerical results, and comparisons with experimental data are all used for model validation.
As an example of validation, consider the modeling of an optical fiber coating applicator. Figure 8 shows the comparison between the numerical results on the velocity distributions with experimental data obtained using a micro PIV system in a transparent system with glycerine as the fluid. Clearly, a fairly good agreement between the experimental and numerical results is observed, lending support to the model. Similarly, the calculated flow

Fig. 6. Calculated velocity and temperature fields in a channel with isolated heat sources and a vortex promoter, simulating an electronic system.

Fig. 7. Calculated flow and temperature distributions in a CVD reactor.

The meniscus at the exit of the applicator is compared with the experimental data in Fig. 9. The meniscus was calculated using the force balance, as mentioned earlier, and a high-speed camera was used to obtain the meniscus experimentally. Again, the comparison is very good. Similarly, other such comparisons are employed to ensure that the model and the simulation are valid and accurate.

Fig. 8. Comparison between calculated and measured velocity profiles in an optical fiber coating process.

Fig. 9. Comparison between calculated and measured exit meniscus in an optical fiber coating process.

FEASIBLE DESIGN

The first step in the design of a thermal system is to obtain an acceptable or feasible design, which satisfies the requirements for the given application, without violating any imposed constraints. Thus, an important consideration is the feasibility of the process, since a fairly narrow domain of operating conditions often arises in which the process is possible. Numerical simulation plays an important role since it can guide the selection of operating conditions and design parameters that can lead to a successful design or process. An example of this consideration is given here.

Using the numerical simulation of the optical fiber drawing process, it can be shown that, for given fiber and preform diameters, it is not possible to draw the fiber at arbitrary furnace wall temperature and draw speed. If the furnace temperature is not high enough, the fiber breaks due to lack of material flow, or viscous rupture. This is indicated by the divergence of the numerical scheme and by excessive tension in the fiber. Similarly, for a given furnace temperature, there is a limit on the speed for drawing, since rupture occurs at higher speeds. The corresponding results are shown in Fig. 10, where a region in which drawing is feasible is identified. For the feasible domain, the draw tension is calculated. The draw tension is small at higher temperatures and lower speeds. Similarly, different combinations of other physical and process variables may be considered to determine the feasible domain.
Once an acceptable design has been obtained, it would generally not be the optimal design, as judged on the basis of cost, performance, performance per unit cost, or some other such measure. The need to optimize has become particularly crucial in recent years due to growing global competition. The optimization process requires the specification of a quantity or function $U$, known as the objective function, which is to be minimized or maximized. If the number of equality constraints $m$ is less than the number of independent variables $n$, an optimization problem is obtained (Jaluria, 1998). The objective function is among the most difficult aspects to be decided in the optimization of thermal systems, since the optimal design is a strong function of the chosen criterion and there may be several important aspects that could be chosen. Optimization is often carried out for different criteria and the final design is chosen by comparison of results for different criteria. Each criterion leads to an optimization curve and the combination of these is known as a Pareto set obtained by considering different criteria in a multi-criteria optimization problem, as discussed in greater detail later. Then, the optimization is based on a trade-off between different criteria.

For thermal systems, the objective function $U$ may be the number of items produced per unit cost, product quality, amount of material produced, energy generated, and so on. Constraints generally arise on the operating and design parameters due to material and system limitations and from conservation principles. For a CVD system, the main qualities of interest include product quality, production rate, and operating cost. These three may be combined into one possible objective function $U$, for example, as (Chiu et al., 2002):

$$U = \frac{(\text{Product Quality}) \times (\text{Operating Cost})}{(\text{Production Rate})}$$

Here, $U$ is to be minimized. The product quality is defined in terms of the maximum variation in film thickness across the susceptor surface, a smaller variation being desirable. The maximum production rate is achieved by placing it in the denominator. The objective function represents equal weighting for each design quality. A minimum value of $U$ implies greater film thickness uniformity. Obviously, the objective function may assume many possible forms. The preceding form is chosen because the magnitude of each aspect does not play a role in locating the optimal solution. It may be factored out as a constant during the minimization process. Another common form of the objective function expresses the effects as a sum or as the square root of the squared sums of the effects. All such possibilities can be employed for the objective function, but the proper form depends on the desired performance of a particular system.

**Response Surfaces**

An approach, which has found widespread use in engineering systems, including thermal systems, is that of response surfaces. The Response Surface Methodology (RSM) comprises of a group of statistical techniques for empirical model building, followed by the use of the model in the design and development of new products and also in the improvement of existing designs (Box and Draper, 1987). RSM is used when only a small number of computational or physical experiments can be conducted due to the high costs (monetary or computational) involved. Response surfaces are fitted to the limited data collected and are used to estimate the location of the optimum. The RSM gives a fast approximation to the model, which can be used to identify important variables, visualize the relationship of the input to the output and quantify trade-offs between multiple...
objectives. This approach has been found to be valuable in developing new processes and systems, optimizing their performance, and improving the design and formulation of new products (Myers and Montgomery, 2002). For example, the relation between the response or output and two design variables \( x_1 \) and \( x_2 \) can be obtained. For each value of \( x_1 \) and \( x_2 \), there is a corresponding value of the response. These values of the response may be perceived as a surface lying above the \( x_1 - x_2 \) plane, as shown in the figures discussed below. It is this graphical perspective of the problem that has led to the term Response Surface Methodology. If there are two design variables, then we have a three-dimensional space in which the coordinate axes represent the response and the two design variables. When there are \( N \) design variables (\( N > 2 \)), we have a response surface in the \( N+1 \) dimensional space. Optimization of the process is quite straightforward if the graphical display could be easily constructed. However, in most practical situations, the true response function is unknown and thus the methodology consists of examining the space of design variables, empirical statistical modeling to develop an approximating relationship (response function) between the response and the design variables, and optimization methods for finding the values of the design variables that produce optimal values of the responses.

The method normally starts with a lower-order model, such as linear or second order. If the second order model is inadequate, as judged by checking against points not used to generate the model, simulations are performed at additional design points and the data used to fit the third order model. Then the resulting third order model is checked against additional data points not used to generate the model. If the third order model is found to be inadequate, then a fourth order model is fitted based on the data from additional simulations and then tested, and so on. A typical second order model for the response, \( z \), is:

\[
Z = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 xy + \beta_4 x^2 + \beta_5 y^2
\]  

Similarly, a third order model for the response, \( z \), is:

\[
Z = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 xy + \beta_4 x^2 + \beta_5 y^2 + \beta_6 x^3 + \beta_7 xy^2 + \beta_8 x^2 y + \beta_9 y^3
\]  

where the \( \beta \)'s are coefficient to be determined from the data and \( x, y \) are the two independent design variables. Once the response surface has been generated, visual inspection can be used to locate the region where the optimum is located and a closer inspection can then be used to accurately determine the location of the optimum. Calculus can also be used to identify minimum or maximum. Both local and global optimum locations can generally be identified. However, since a limited number of data points are used in order to generate the response surface, the surface approximates the actual behavior and the results are similarly approximate, though for many practical problems this is quite adequate.

A Compromise Response Surface Method (CRSM), which is a modification of the response surface method, has been developed to reduce the simulation runs needed for generating the response surface and thus increase the efficiency of the optimization strategy (George and Ogot, 2005). This approach has been used for an impingement CVD reactor for thin film fabrication. Figure 11 shows a typical response surface obtained for the uniformity parameter, which is defined in terms of the variation in film thickness across the susceptor. Better uniformity is indicated by a smaller uniformity parameter. Second order response surfaces are used to model the deposition rate and the uniformity of the film thickness, using the inputs from the simulation. The resulting relationships are used to optimize the operating conditions, such as the susceptor temperature and flow rate.

Fig. 11. Response surfaces for an impingement CVD reactor for depositing Silicon.

Conventional approaches for design and optimization are based on sequential use of numerical simulation and experiment. Thus, they fail to use certain advantages of using both tools concurrently. Recent study has aimed at combining simulation and experiment in a concurrent manner such that outputs of each approach drives
the other to achieve better engineering design in a more efficient way. This concept was employed for the design of a cooling system for electronic equipment, which consists of multiple heat sources in a channel, as shown in Fig. 3. The effects of different coolants, materials and geometry were obtained by numerical simulation, because of ease of varying these, and the effects of heat input and flow rate by experiment, since these were relatively easily varied experimentally. Switching from one approach to the other was based on flow transition to turbulence and on the design parameter being varied (Icoz and Jaluria, 2004).

The results from numerical simulation, as well as from experimentation, can be used to generate response surfaces for the heat transfer rates from the heat sources in Fig. 3 and for the pressure drop as functions of the design variables, which include the geometry parameters, fluid, flow rates, materials, etc. Response surfaces of different orders can be generated, though lower order surfaces are desirable for ease in optimization. Several second and third order response surfaces were obtained for the electronic cooling problem of Fig. 3, with the variables normalized for greater accuracy in the calculations. Once the response surfaces are found for individual objectives, they can be combined to form a single objective function, as mentioned earlier. Figure 12 shows the response surfaces in terms of the heat transfer rates from the two sources, as well as the pressure, with the Reynolds number $Re$, and the vortex promoter dimensions $h_p/H$ as the independent variables for different geometries of the promoter. Also, monotonic variations are often seen, indicating optima at the boundaries.

Search Methods

Search methods constitute the most important optimization strategy for thermal systems. Many different approaches have been developed and are particularly appropriate for specific problems. The underlying idea is to generate a number of designs, which are also called trials or iterations, and to select the best among these. Effort is made to keep the number of trials small, often going to the next iteration only if necessary. This is an important consideration with respect to thermal systems since each trial may take a considerable amount of computational or experimental effort. The steepest ascent/descent method and other gradient-based methods are widely used for thermal systems. Many of these are hill-climbing techniques in that they attempt to move towards the peak, for maximizing the objective function, or towards the valley, for minimizing the objective function, over the shortest possible path, as outlined by Haug and Arora (1979) and Arora (1989). At each step, starting with the initial trial point, the direction in which the objective function changes at the greatest rate is
chosen for moving the location of the point which represents the design on the multivariable space. The direction of the gradient vector $\nabla U$ is used since it is the direction in which $U$ changes at the greatest rate. Thus, the number of trial runs needed to reach the optimum is kept small. However, an evaluation of the gradients is needed to determine the appropriate direction of motion, limiting the application of the methods to problems where the gradients can be obtained accurately and easily. Numerical differentiation is generally needed for thermal systems, which are usually governed by nonlinear equations. Genetic optimization algorithms, that are based on function evaluations instead of gradients, have also been developed and used (Goldberg, 1989). These methods are based on inheritance of characteristics and learning, as derived from human genetics.

Figures 13 and 14 show the results for the optical fiber drawing process, considering the numerical simulation of the draw furnace, and for multiple heat sources in a channel. Because of the complexity of the optical fiber drawing process and lack of information on operating costs, the effort was directed at the fiber quality, taking the tension, defect concentration and velocity difference across the fiber as the main considerations, all these being scaled to obtain similar ranges of variation. The objective function $U$ was taken as the square root of the sum of the squares of these three quantities and was minimized. Several search methods, such as golden-section for single variable and univariate for multivariable cases, were employed. Figure 13 shows typical results from golden-section search for the optimal draw temperature. The results from the first search are used in the second search, following the univariate search strategy, to obtain optimal design in terms of these two variables. Several other results were obtained on this complicated problem, particularly on furnace dimensions and operating conditions to achieve optimal drawing. Figure 14 shows the results for an objective function that combines the heat transfer rate and the pressure head needed for the flow in the channel. Thus, two objective functions are combined into one to obtain the resulting optimum.

![Fig. 13. Optimization of fiber quality with draw furnace temperature as the dominant variable.](image1)

![Fig. 14. Optimization of the electronic cooling problem of Fig. 3, using a combination of heat transfer rate and pressure as the objective function.](image2)

**Multi-Objective Optimization**

It has been mentioned earlier that optimal conditions are generally strongly dependent on the chosen objective function. However, as discussed in the preceding section, not one but several features or aspects are typically important in most practical applications. In thermal systems, the efficiency, production rate, output, quality, and heat transfer rate are common quantities that are to be maximized, while cost, input, environmental effect, and pressure are quantities that need to be minimized. Thus, any of these could be individually chosen as the objective function, though interest clearly lies in dealing with more than one objective function. The use of the trade-off curve was outlined earlier.

A common approach to multiple objective functions is to combine them to yield a single objective function that is minimized or maximized. Examples given earlier include output/cost, quality/cost and efficiency/input. In heat exchangers and cooling systems for electronic equipment, it is desirable to maximize the heat transfer rate. But this comes at the cost of flow rate or pressure head. Then heat transfer rate/pressure head could be chosen as the objective function, see Fig. 14. Similarly, additional aspects could be combined to obtain a single objective function, e.g., objective function $U = (\text{quality}) \times (\text{production rate})/(\text{cost})$. However, the various quantities that compose the objective function should be scaled and weighted in order to base the system optimization on the importance of each in comparison to the others. For instance, heat transfer rate and pressure head may be scaled with the expected maximum values in a given instance so that both vary from 0 to 1. Other nondimensionalizations are also possible, to ensure that equal importance is given to each of these. Weights can similarly be used to increase or decrease the importance of a given quantity compared to the others. Derived
quantities like logarithm or exponential of given physical quantities may also be employed for scaling and for considering appropriate ranges of the quantities. Clearly, the objective function thus obtained is not unique and different formulations can be used to generate different functions, which could presumably yield different optima.

Another approach, which has gained interest in recent years, is that of multi-objective optimization. In this case, two or more objective functions that are of interest in a given problem are considered and a strategy is developed to trade-off one objective function in comparison to the others (Miettinen, 1999; Deb, 2001). Let us consider a problem with two objective functions \( f_1 \) and \( f_2 \). With no loss of generality, we can assume that each of these is to be minimized, since maximization is equivalent to minimization of the negative of the function. The values of \( f_1 \) and \( f_2 \) are shown for 5 designs in Fig. 15, each design being indicated by a point. Design 2 dominates Design 4, since both objective functions are smaller for Design 2 compared to Design 4. Similarly, Design 3 dominates Design 5. However, Designs 1, 2 and 3 are not dominated by any other design. The selection of the better design is straightforward for the dominated cases, though not so for the others. The set of non-dominated designs is termed the Pareto Set, which represents the best collection of designs. As shown in Fig. 15 (b) the near horizontal or near vertical sections are omitted to obtain proper efficiency for design selection and a Pareto Front is obtained. Then, for any design in the Pareto Set, one objective function can be improved, i.e., reduced as considered here, at the expense of the other objective function. The same arguments apply for more than two objective functions. The set of designs that constitute the Pareto Set represent the formal solution in the design space to the multi-objective optimization problem. The selection of a specific design from the Pareto Set is left to the decision maker or the engineer. A large literature exists on utility theory, which seeks to provide additional insight to the decision maker to assist in selecting a specific design, see Ringuest (1992). For different concepts, such as geometrical configurations, different Pareto Fronts can be generated, with the envelope of these yielding the desired solution. Many multi-objective optimization methods are available that can be used to generate Pareto solutions. Various quality metrics are often used to evaluate the “goodness” of a Pareto solution obtained and possibly improve the method as well as the optimal solution.

The electronic cooling system can also be considered without the vortex promoter. The total heat transfer rate and the pressure head are taken as the two main objective functions. Response surfaces can be drawn for these to investigate the optimum. Multi-objective function optimization can also be used, as discussed above. Both experimental and computation data are used to build the data base for the response surfaces. Second order, third order and higher order regression models are considered. Comparing the second order with the third order, it was observed that the third order fitting was substantially a better choice since it had higher correlation coefficients. The difference between third order and fourth order models was small. Hence, the third order model, based on computational and experiment data, was employed as the regression model for the multi-objective design optimization problem. The response surfaces obtained from this regression model for \( \Delta P \) and the total heat transfer rate, given in terms of the Stanton number, \( St \), where \( St \) is the Nusselt number divided by the Reynolds and Prandtl numbers, are shown in Fig. 16 (a) and (b), respectively. After the regression model is obtained for \( \Delta P \) and \( St \), the Pareto set is obtained for the multi-objective design optimization problem (Zhao, et al., 2007). The resulting Pareto set is plotted in Fig. 16 (c). From the figure, it is observed that if the pressure drop is decreased, implying a lower flow rate, the Stanton number is also decreased, and vice versa. The maximum Stanton number and the minimum pressure drop cannot be obtained at the same time. This is expected from the discussion of the physical problem given earlier. A higher heat transfer rate requires a greater flow rate, which in turn needs a greater pressure head. Therefore, interest lies in maximizing heat transfer and minimizing the pressure head. As for decision-making, other considerations have to be added to select the proper solution from the Pareto set, as outlined earlier.
CONCLUSIONS

Important considerations in the numerical simulation, design and optimization of thermal systems are presented in this paper. Simulation generally involves complicated domains, large property changes, combined mechanisms and intricate boundary conditions. The complexity introduced due to these and the stringent demands placed on the numerical scheme, particularly in terms of the nonlinearity and coupling of the governing equations, are discussed. The validation of the model, in terms of existing results and new experimental data, is outlined. The determination of a feasible or acceptable design on the basis of simulation is discussed. Optimization is presented in terms of conventional approaches, as well as recent ones using concurrent simulation and experimentation, response surfaces and multi-objective optimization. Results for a few important practical problems are presented to illustrate the use of these approaches.

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Fig. 16. Response surfaces and the Pareto set for the electronic cooling system.

NOMENCLATURE

\( C_p \) \quad \text{specific heat at constant pressure, J/kgK}

\( h \) \quad \text{height of heat sources, m}

\( h_p \) \quad \text{dimension of vortex promoter, m}

\( H \) \quad \text{channel height, m}

\( K \) \quad \text{thermal conductivity, W/mK}

\( L \) \quad \text{characteristic length, m}

\( p \) \quad \text{local pressure, N/m}^2

\( Pr \) \quad \text{Prandtl number, dimensionless}

\( Q \) \quad \text{heat transfer rate per unit length, W/m}

\( R \) \quad \text{radial coordinate distance, m}

\( Re \) \quad \text{Reynolds number, dimensionless}

\( S_r \) \quad \text{radiative source, J/m}^3\text{s}

\( St \) \quad \text{Stanton number, dimensionless}

\( t \) \quad \text{time, s}

\( T \) \quad \text{temperature, K}

\( T_{\text{melt}} \) \quad \text{glass softening temperature, K}

\( u, v \) \quad \text{velocity components, m/s}

\( U \) \quad \text{objective function}

\( x, y \) \quad \text{coordinate distances, m}

\( z \) \quad \text{axial coordinate, m}

\textbf{Greek Symbols}

\( \lambda \) \quad \text{wavelength, \( \mu \text{m} \)}
\( \mu \) dynamic viscosity of fluid, kg/ms  
\( \nu \) kinematic viscosity, m²/s  
\( \Phi \) viscous dissipation function, J/m³s  
\( \rho \) density, kg/m³  
\( \theta \) dimensionless temperature

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