Stresses Around an Elliptical Hole in a Finite Plate Subjected to Axial Loading

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A photelastic solution of the distribution of stresses around a centrally located elliptical hole in a plate of finite width subjected to uniform axial loading is presented in this paper. Stress distributions at the boundaries are given for a wide range of the parameters, and stress-concentration factors have been computed for the points of maximum tensile and of maximum compressive stress. Comparisons are made with available theoretical solutions. Some appreciable discrepancies are pointed out.


The experiments reported in this paper deal with sixteen models represented in Fig. 1. Each model is a long plate or strip, subjected to uniformly distributed tensile load at the enlarged end with a central elliptical hole, Fig. 2. The major axis of elliptical hole is either aligned with, or perpendicular to, the direction of the long strip.

The widths of the models and the ellipses will be described by the dimensionless terms \( \lambda \) and \( \mu \) where

\[
\lambda = \frac{2a}{W}, \quad \mu = \frac{b}{a}
\]

Fig. 1 Geometry and loading of sixteen strips, with central elliptical holes, under axial load

Fig. 2 Samples of isochromatic fringe patterns of four different ellipses under axial load
and $W = \text{model width}$

$2a = \text{transverse axis of the ellipse}$

$2b = \text{longitudinal axis of the ellipse}$

Special interest exists in the determination of the stresses present at the critical points, $A$ and $B$, where maximum tension and maximum compression occur, Fig. 1. The investigation also included the determination of the stress distribution at the boundaries of the hole for several plate widths and hole ellipticity. The results are given in dimensionless form as stress ratios and stress-concentration factors.

The sixteen models were analyzed with the usual two-dimensional photoelasticity method. The maximum fringe orders $n_{\text{max}}$, obtained at several load levels, were plotted against the corresponding average fringe order $n_{\text{ave}}$. The value of $n_{\text{ave}}$ is the average of the fringes (obtained by compensation) in the calibration zone. A straight line was drawn through the points to obtain the $n_{\text{max}}/n_{\text{ave}}$ ratio. Any possible initial or residual stress present in the model is corrected by following this procedure.

The curves of stress-concentration factor $K = \sigma_{\text{max}}/\sigma_{\text{ave}} = n_{\text{max}}/n_{\text{ave}}$, plotted as a function of $\lambda$, are shown in Figs. 3, 4, and 5. The known solutions for a circular hole in an infinite and in a

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**Fig. 3** Stress-concentration factors ($K$) for points under maximum tension in finite-width plate with an elliptical hole

**Fig. 4** Stress-concentration factors ($K$) for points under maximum compression in finite-width plate with an elliptical hole

**Fig. 5** Stress-concentration factors ($K$) for points under maximum compression using $\lambda = 2a/W$ as parameter
finite plate, established by G. Kirsch [6], R. Howland [7], and A. Wahl and R. Beeuwkes [8], are added for completeness. Inglis' [1] theoretical values of stress concentrations in a plate of infinite width are also shown:

\[ K = 1 + 2 \frac{a}{b} \text{ at point } A \]

\[ K = -1 \text{ at point } B \]

These \( K \)-factors completely define the maximum tensile and compressive stresses in the models. It may be observed that, as the width of the plate approaches the size \( 2a \) of the elliptical hole, Fig. 1, the \( K \)-factors of points \( A-A \) for maximum tension approach infinity. This sharp increase is due to the reduction of area and does not show well the influence of the geometry of the elliptical hole. Another way to define the stress-concentration factor which would emphasize the influence of the geometry of the elliptical hole consists in the use of an average stress \( \sigma_{av} \), computed over the width \( W' \), where \( W' \) is the total width of the strip \( W \) minus the length \( 2a \) of the elliptical hole (Fig. 1).

Call this second stress-concentration factor \( K' \) (\( K' = \sigma_{max}/\sigma_{av} \)).

The curves of \( K' \) as a function of \( \lambda \) are shown in Fig. 6.

Figs. 7, 8, 9, and 10 show the stress distribution around the boundary of the elliptical hole.

It should be noted that when the minor axis of the elliptical hole is aligned with the long centerline of the strip and the curva-
ture at the end of the major axis is very small, then even a very small experimental deviation in the desired curvature of the model would greatly affect the stress-concentration factors. To increase the precision of the determination, a larger sized model was used for the thirteenth through the sixteenth tests.

As the width of the testing specimen becomes narrower, the stress-concentration factor ($K$) increases over the whole range of $\lambda$. Only one exception to the rule has been obtained, and this occurred in the thirteenth model.

Curves of stress-concentration factor ($K$) (or $K'$) versus $\lambda$ (using $\mu$ as a parameter) show an appreciable deviation from the curves plotted using H. Neuber’s approximate theoretical solution. Sometimes the difference is of the order of 25 percent.

Fig. 5 is included to indicate the influence of the parameter $\lambda$ on the compressive stress-concentration factors.

References