Repetitive laser pulse heating analysis: Pulse parameter variation effects on closed form solution

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Abstract

Laser conduction limited heating finds wide application in surface processing industry. Modelling of the heating process gives insight into the physical processes involved in conduction limited heating. Moreover, analytical solutions provide functional relation among the parameters that influence the heating process. In the present study, repetitive laser pulse heating of a solid substrate is considered. A closed form solution for the temperature rise including the cooling cycle is obtained using a Laplace transformation method. It is found that the results obtained from the closed form solution agree well with the numerical predictions. The maximum surface temperature rises rapidly once the cooling period between the consecutive pulses reduces.

Keywords: Laser; Conduction; Pulse heating; Closed form

1. Introduction

Laser repetitive pulse heating process finds wide application in industry. Since lasers deliver high intensity beam onto the substrate surface, a local thermal processing is possible. Irradiation of a surface with a single laser pulse does not provide sufficient energy deposited onto the surface. Consequently, repetitive pulse irradiation becomes inevitable during the laser heating process. Moreover, a closed form solution for the temperature rise provides useful information on the heating parameters and laser pulse properties. In addition, model studies reduce the experimental cost and minimizes the experimentation time.

Laser pulse heating falls into conduction and non-conduction limited heating processes. In the case of conduction limited heating process, the substrate material remains in solid phase while phase change occurs in non-conduction limited heating process. Considerable research studies were carried out to explore the laser conduction heating process. An analytical solution for a laser step input pulse intensity was obtained by Ready [1]. Simon et al. [2] studied the
heat conduction in deep penetration welding and showed that the time modulated laser beam had little effect on the resulting heat affected zone. Laser heating of a two-layer system was studied by Al-Adawi et al. [3] using a Laplace transformation method. They indicated that the time to reach the melting of the substrate material was dependent highly on the thermal properties. Blackwell [4] studied analytically the temperature rise due to a laser heating pulse and convective boundary condition at the surface. He obtained a closed form solution for the temperature rise inside the substrate material. Thermal analysis of a surface transformation hardening due to laser pulse heating was carried out by Woodard and Dryden [5]. An analytical solution for the axisymmetric heat conduction was presented. Thermal analysis of laser heat treatment of engineering alloys was studied by Yilbas et al. [6]. They obtained a closed form solution for temperature rise and developed a relation between temperature rise and equilibrium time. An analytical solution for a laser pulse input pulse heating was obtained by Yilbas and Shuja [7]. They introduced a power relation between the dimensionless temperature and distance. An analytical solution for a repetitive laser pulse heating with a convective cooling boundary condition at the surface was obtained by Yilbas and Kalyon [8]. They introduced a time exponentially varying pulse in the analysis. When formulating the laser conduction limited heating process, the cooling cycle onset of laser pulse ending should be taken into account. Moreover, in repetitive laser pulse heating process, the period (cooling period) between the successive pulses should be considered. Consequently, the analysis of repetitive laser pulse heating employing the cooling period between the consecutive pulses becomes necessary.

In the present study, an analytical solution for the laser repetitive pulse heating process is considered. A closed form solution for the temperature rise due to laser repetitive pulses is obtained using a Laplace transformation method. The results obtained from the closed solution is compared with the numerical predictions. It should be noted that in order to obtain a closed form solution for the temperature distribution independent of material and laser pulse properties, the Fourier heat transfer equation is non-dimensionalized and the results are presented in dimensionless form. Consequently, the results obtained can be changed into the dimensional form through accommodating the material properties and pulse intensity in the dimensionless time, space and temperature.

2. Mathematical modelling

The heat transfer equation for a laser heating pulse can be written as

\[ \frac{\partial T}{\partial t} = \alpha \nabla^2 T + \bar{T}(x,t) \]

where \( T \) is the temperature in the heat affected zone, \( \alpha \) is the thermal diffusivity, \( \nabla^2 \) is the Laplacian operator, and \( \bar{T}(x,t) \) is the temperature in the Laplace domain.

<table>
<thead>
<tr>
<th>Nomenclature</th>
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<tbody>
<tr>
<td>( C_p )</td>
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<tr>
<td>( I_1 )</td>
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<tr>
<td>( I_0 )</td>
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<td>( d_r^* )</td>
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<td>( d_r^*_2 )</td>
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<td>( T(x,t) )</td>
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<td>( \bar{T} )</td>
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<td>( x )</td>
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<td>( x^* )</td>
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Greek letters

| \( \alpha \) | thermal diffusivity \( (m^2/s) \) |
| \( \delta \) | absorption coefficient \( (1/m) \) |
| \( \Delta \) | shift in time |
| \( \rho \) | density \( (kg/m^3) \) |
\[
\frac{\partial^2 T}{\partial x^2} + \frac{I_1 \delta}{k} e^{-\alpha t} (C_1 P_1 + C_2 P_2) = \frac{\partial T}{\partial t}
\]  
(1)

where \( I_1 = (1 - r_1) I_0 \) and \( P_1 = 1[t - t_1] - 1[t - t_1 + dt_1], P_2 = 1[t - t_2] - 1[t - (t_2 + dt_2)]. \)

\[
1[t] = \begin{cases} 
1, & t > 0 \\
0, & t < 0 
\end{cases} \quad 1[t - t_0] = \begin{cases} 
1, & t > t_0 \\
0, & t < t_0 
\end{cases} 
\]  
(2)

where \( r_1 \) is the reflection coefficient and \( I_0 \) is the peak power intensity. \( P_1 \) and \( P_2 \) are the step input intensity functions for the first and second consecutive pulses, \( 1[t] \) is the unit step function and \( 1[t - t_0] \) is the shifted unit step function (shifted by \( t_0 \)), \( t_1 \) and \( t_2 \) are first and second pulse initiations, \( dt_1 \) and \( dt_2 \) are the first and second pulse lengths, and \( C_1 \) and \( C_2 \) are the constants defining the peak amplitude of pulses \( (0 \leq C_1 \leq 1 \) and \( 0 \leq C_2 \leq 1) \) (Fig. 1a). Therefore, the amount of time shift between two consecutive pulses is \( t_2 - (t_1 + dt_1). \)

The initial and boundary conditions are:

At time \( t = 0 \): \( T(x, 0) = T_0, \) \( \frac{\partial T}{\partial x}|_{x=0} = 0 \) and at \( x = \infty \): \( T(\infty, t) = T_0 \)

In order to generalize the solution independent of material and laser pulse properties, which could be applicable to any material, non-dimensionalizing of the parameters in the Fourier heat transfer equation (Eq. (1)) are essential. After introducing the dimensionless parameters as

\[
t^* = \alpha \delta t, \quad x^* = \delta x, \quad T^* = \frac{T k \delta}{I_1}
\]

Eq. (1) becomes

\[
\frac{\partial^2 T^*}{\partial x^2} + [C_1 P_1^* + C_2 P_2^*] e^{-\alpha x^*} = \frac{\partial T^*}{\partial t^*}
\]  
(3)

where

\[
P_1^* = 1[t^* - t_1^*] - 1[t^* - t_1^* + dt_1^*], \\
P_2^* = 1[t^* - t_2^*] - 1[t^* - t_2^* + dt_2^*]
\]

where \( t_1^* = \alpha \delta^2 t_1, \) \( t_2^* = \alpha \delta^2 t_2, \) \( dt_1^* = \alpha \delta^2 dt_1, \) and \( dt_2^* = \alpha \delta^2 dt_2. \)

The non-dimensional initial and boundary conditions become:

At time \( t^* = 0 \): \( T^*(x^*, 0) = 0, \) \( \frac{\partial T^*}{\partial x}|_{x^*=0} = 0 \) and at \( x^* = \infty \): \( T^*(\infty, t^*) = T_0^* \)

The solution of Eq. (3) can be obtained through Laplace transformation method, i.e., with respect to \( t^*, \) the Laplace transformation of Eq. (3) yields:

\[
\frac{\partial^2 T^*}{\partial x^2} + [C_1 P_1^* + C_2 P_2^*] e^{-\alpha x^*} = \frac{\partial T^*}{\partial t^*}
\]

or

\[
\frac{\partial^2 T^*}{\partial x^2} - s T^* = -[C_1 P_1^* + C_2 P_2^*] e^{-\alpha x^*} - T_0^*
\]

(5)

The homogenous solution of Eq. (5) can be written as

\[
T_{h}^* = K_1 e^{\sqrt{s} t^*} + K_2 e^{-\sqrt{s} t^*}
\]

(6)

and the particular solution for Eq. (5) is

\[
T_{p}^* = \frac{C_1 P_1^* + C_2 P_2^*}{s - 1} e^{-\alpha x^*} + \frac{T_0^*}{s}
\]

(7)

Therefore, the solution of dimensionless temperature in the Laplace domain becomes

\[
T^* = K_1 e^{\sqrt{s} t^*} + K_2 e^{-\sqrt{s} t^*} + \frac{C_1 P_1^* + C_2 P_2^*}{s - 1} e^{-\alpha x^*} + \frac{T_0^*}{s}
\]

(8)

The boundary condition \( T(\infty, t^*) = T_0^* \) results in \( K_1 = 0. \) The boundary condition at the surface yields

\[
\left[ \frac{\partial T^*}{\partial x} \right]_{x^*=0} = -K_2 e^{-\alpha x^*} \sqrt{s}
\]

\[
- \frac{C_1 P_1^* + C_2 P_2^*}{s - 1} e^{-\alpha x^*} \right|_{x^*=0} = 0
\]

which results

\[
K_2 = -\frac{C_1 P_1^* + C_2 P_2^*}{\sqrt{s}(s - 1)}
\]
Therefore, Eq. (8) becomes
\[ \bar{T} = C_1 \frac{P_1(s) + C_2 P_2(s)}{(s-1)} e^{-sT} + \frac{C_1}{s} e^{-sT} + \frac{T_0}{s} \]
Noting from Eq. (4) that:
\[ P_1(s) = \frac{e^{-sT}}{s} - \frac{e^{-(s+1)T}}{s}, \]
\[ P_1(s) = \frac{e^{-sT}}{s} - \frac{e^{-(s+1)T}}{s} \]
Hence Eq. (9) becomes:
\[ \bar{T} = - \left\{ \frac{C_1}{s} e^{-sT} + \frac{C_1}{s} e^{-sT} + \frac{T_0}{s} \right\} \]
The inversion of Laplace transforms can be written as
\[ \mathcal{L}^{-1}\left[ \frac{e^{-\sqrt{s}t}}{s\sqrt{s}(s-1)} \right] = \mathcal{L}^{-1}\left[ \frac{e^{-\sqrt{s}t}}{\sqrt{s}(s-1)} \right] - \mathcal{L}^{-1}\left[ \frac{e^{-\sqrt{s}t}}{s} \right]. \]

Consequently, Laplace transformation of \[ \mathcal{L}^{-1}\left[ \frac{e^{-\sqrt{s}t}}{s\sqrt{s}(s-1)} \right] \] yields [9]:
\[ \mathcal{L}^{-1}\left[ \frac{e^{-\sqrt{s}t}}{s\sqrt{s}(s-1)} \right] = \frac{e^t}{2} - e^t \cdot \text{Erfc} \left( \sqrt{t} + \frac{x^t}{2\sqrt{t}} \right) - e^t \cdot \text{Erfc} \left( \sqrt{t} - \frac{x^t}{2\sqrt{t}} \right) - \frac{2\sqrt{t^r} e^{-x^t/4t^r}}{\sqrt{\pi}} - x^t \cdot \text{Erfc} \left( \frac{x^t}{2\sqrt{t^r}} \right). \]

Consider the following Laplace operation:
\[ \mathcal{L}^{-1} e^{-\Delta t} F(s) = f(t - \Delta t) \]
where \( \mathcal{L}^{-1} F(s) = f(t) \).

Therefore
\[ \mathcal{L}^{-1} e^{-\Delta t} \left[ \frac{e^{-\sqrt{s}t}}{s\sqrt{s}(s-1)} \right] = f(x^t, t^r) \]

More explicitly:
\[ \mathcal{L}^{-1} \left[ \frac{e^{-\sqrt{s}t}}{s\sqrt{s}(s-1)} \right] = \frac{e^{t-r^r}}{2} \left[ e^{-x^t} \cdot \text{Erfc} \left( \sqrt{t^r} + \frac{x^t}{2\sqrt{t^r}} - \Delta \right) - e^{-x^t} \cdot \text{Erfc} \left( \sqrt{t^r} - \frac{x^t}{2\sqrt{t^r}} - \Delta \right) - \frac{2\sqrt{t^r} e^{-x^t/4(t^r - \Delta)}}{\sqrt{\pi}} - x^t \cdot \text{Erfc} \left( \frac{x^t}{2\sqrt{t^r - \Delta}} \right) \right] = H_A \]

where \( \Delta \) is amount of shift in time due to two consecutive pulses.

Laplace transformation of \[ \mathcal{L}^{-1}\left[ \frac{e^{-\sqrt{s}t}}{s(s-1)} \right] \] yields
\[ \mathcal{L}^{-1}\left[ \frac{e^{-\sqrt{s}t}}{s(s-1)} \right] = e^{-x^t}[e^{t^r} - 1(t^r)] = H_B \]

where \( 1(t^r) \) is a unit step function. Similarly,
\[ \mathcal{L}^{-1}\left[ \frac{e^{-\Delta t} e^{-\sqrt{s}t}}{s\sqrt{s}(s-1)} \right] = \mathcal{L}^{-1}\left[ \frac{e^{-\sqrt{s}t}}{s\sqrt{s}(s-1)} \right] \]

Moreover, we note from Eqs. (10) and (11) that:
\[ \mathcal{L}^{-1} e^{-\Delta t} \left[ \frac{e^{-\sqrt{s}t}}{s\sqrt{s}(s-1)} \right] = \text{Term1}_A = H_A(\Delta)\big|_{\Delta = t^r}, \]
\[ \mathcal{L}^{-1} e^{-\Delta t} \left[ \frac{e^{-\sqrt{s}t}}{s\sqrt{s}(s-1)} \right] = \text{Term2}_A = H_A(\Delta)\big|_{\Delta = t^r}, \]
\[ \mathcal{L}^{-1} e^{-\Delta t} \left[ \frac{e^{-\sqrt{s}t}}{s\sqrt{s}(s-1)} \right] = \text{Term3}_A = H_A(\Delta)\big|_{\Delta = t^r}, \]
\[ \mathcal{L}^{-1} e^{-\Delta t} \left[ \frac{e^{-\sqrt{s}t}}{s\sqrt{s}(s-1)} \right] = \text{Term4}_A = H_A(\Delta)\big|_{\Delta = t^r + d^r}. \]

Similarly, the following terms can be written:
\[ \mathcal{L}^{-1} e^{-\Delta t} \left[ \frac{e^{-\sqrt{s}t}}{s(s-1)} \right] = \text{Term1}_B = H_B(\Delta)\big|_{\Delta = t^r}, \]
\[ \mathcal{L}^{-1} e^{-\Delta t} \left[ \frac{e^{-\sqrt{s}t}}{s(s-1)} \right] = \text{Term2}_B = H_B(\Delta)\big|_{\Delta = t^r + d^r}, \]
\[ \mathcal{L}^{-1} e^{-\Delta t} \left[ \frac{e^{-\sqrt{s}t}}{s(s-1)} \right] = \text{Term3}_B = H_B(\Delta)\big|_{\Delta = t^r}, \]
\[ \mathcal{L}^{-1} e^{-\Delta t} \left[ \frac{e^{-\sqrt{s}t}}{s(s-1)} \right] = \text{Term4}_B = H_B(\Delta)\big|_{\Delta = t^r + d^r}. \]

Using the terms in equations (13) and (14), dimensionless temperature (Laplace inversion of Eq. (10)) becomes:
Pulse properties used in the present study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tr>
<td>$t_1^*$</td>
<td>0</td>
</tr>
<tr>
<td>$t_2^*$</td>
<td>5–15</td>
</tr>
<tr>
<td>$d_1^*$</td>
<td>5</td>
</tr>
<tr>
<td>$C_1$</td>
<td>1</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 1b, i.e., the first unit step input pulse starts at time $t_1$ while the second unit step input pulse starts at time $t_1 + d_1^*$. The difference in both pulses results in the first pulse of the consecutive pulses, i.e.:

$$P_1(t) = 1[t - t_1] - 1[t - (t_1 + d_1^*)]$$

Fig. 1b, i.e., the first unit step input pulse starts at time $t_1$ while the second unit step input pulse starts at time $t_1 + d_1^*$. The difference in both pulses results in the first pulse of the consecutive pulses, i.e.:

$$P_1(t) = 1[t - t_1] - 1[t - (t_1 + d_1^*)]$$

where $1[t - t_1]$ and $1[t - (t_1 + d_1^*)]$ represent the shifted unit step functions with shifted amounts of $t_1$ and $d_1^*$, respectively. These unit step functions are defined in Eq. (2). Similar arguments are applied to the second pulse of the consecutive pulses when constructing it, i.e.:

$$P_2(t) = 1[t - t_2] - 1[t - (t_2 + d_2^*)]$$

Moreover, the pulse intensity of consecutive pulses can be varied by introducing the intensity multiplication factors for each pulse of the consecutive pulses, i.e.:

$$C_1P_1(t) + C_2P_2(t)$$

where $C_1$ and $C_2$ are intensity multiplication factors where $0 < C_1 < 1$ and $0 < C_2 < 1$. Table 1 gives the properties of different pulse types. In order to obtain a general solution for the temperature rise, the Fourier heating equation is non-dimensionalized. This provides the solution independent of material properties and laser pulse intensity. Consequently, the closed form solution for temperature distribution can be used for any substrate material. One of the examples of such situation is given in Fig. 2, i.e. temporal variation of the surface temperature distribution for steel is shown for a single pulse with pulse length $1.4 \times 10^{-8}$ s and pulse intensity $2 \times 10^{11}$ W/m$^2$.

Fig. 2 shows the temporal variation of surface temperature obtained from the closed form solution and numerical simulations. The temperature profiles obtained from both analysis are in excellent agreement.

Fig. 3a shows the temporal variation of non-dimensional temperature distribution at different locations inside the substrate material. Temperature rises rapidly in the beginning of the heating pulse and as the heating pulse progresses the rate of temperature...
profile rise reduces slightly. This can also be seen from Fig. 3b, in which time derivative of temperature gradient is shown. This indicates that in the early heating period, the internal energy gain of the substrate material due to absorption of irradiated energy dominates over the conduction energy transport from surface vicinity to the solid bulk. As the heating period progresses, temperature gradient increases and heat conduction from the surface vicinity to the solid bulk enhances. The material response to a heating pulse at some depth below the surface differs. In this case, temperature profiles becomes almost similar in depths similar to absorption depth of the substrate material ($x^* < 1$). As the depth increases further from $x^* = 1$, the energy gain due to absorption becomes almost negligible and energy transport by conduction is almost the sole mechanism governing the temperature rise in this region. Consequently, material response to a heating pulse slows down and the rate of temperature rise reduces in this region. The influence of the second pulse on temperature profiles is similar at the surface as well as inside the substrate material, i.e. magnitude of temperature rise in the surface region is similar to that corresponding to some depth below the surface, provided that $x^* < 1$. Moreover, the domination of internal energy gain over the conduction energy transport in the surface region during the early heating period is not observed at some depth below the surface. This is because of the energy transfer mechanism, in which case, absorption replaces with the conduction heating in this region.

Figs. 4 and 5 show temporal variation of non-dimensional temperature profiles corresponding to
other two pulse types employed in the simulations (Table 1). It should be noted that the duration (cooling period) between two pulses is reduced for pulse type 2 and the cooling period is set to zero for pulse type 3. The later (pulse type 3) provides the basis for comparison of temperature profiles due to a single pulse with double pulse intensity and pulse type 3. Temperature profiles show similar trends to those shown in Fig. 3a, provided that the magnitude of maximum temperature increases considerably after the second pulse arrives. This occurs because of the cooling period between the pulses, which is short for pulse type 2, i.e. temperature inside the substrate material remains high after the first pulse when the second pulse is initiated. Consequently, high base temperature results in the attainment of a high maximum temperature after the arrival of the second pulse. In the case of non-existing cooling period (pulse type 3), temperature profile rises continuously, i.e. the second pulse is triggered immediately after the first pulse ends. Although the material response to the second pulse results in gradual rise of temperature during the early heating period of the second pulse, the rise of temperature is not discontinued, i.e. the rate of temperature rise changes gradually due to two pulses. Material response to a single pulse with double intensity becomes significant when comparing its counterpart due to two pulses with zero cooling period. In this case, the rate of temperature rise is considerably higher than the consecutive two pulses with no cooling period between the pulses.

4. Conclusions

An analytical solution for repetitive laser pulse heating of solid substrate is considered and a closed form solution for temperature rise inside the material is obtained using a Laplace transformation method. The governing equation of heat transfer is non-dimensionalized to obtain a closed form solution independent of material and laser pulse properties and the influence of the period (cooling period) between two successive pulses on the temperature rise is investigated. It is found that the magnitude of the maximum surface temperature is influenced by the cooling period of two successive pulses. The rapid response of material to heating pulses is more pronounced in the region just below the surface \((x^* < 1)\). The specific conclusions derived from the present study can be listed as follows:

1. Temperature profiles obtained from the closed form solution agrees well with the numerical predictions. Analytical solution provides the temperature distribution in the heating cycle as well as in the cooling cycle of the heating pulse (after the pulse ends).
2. The rate of temperature rise in the early heating period is higher than that corresponding to longer heating periods. In this case, internal energy gain dominates over the conduction energy transfer from surface vicinity to the solid bulk of the substrate material.
3. Temperature rise due to consecutive two pulses without having the cooling period between them is less than that corresponding to a single pulse with double intensity.

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