Chapter 1
Introduction

1  Overview of the Course

This is a course about the mechanical response of structural materials to the application of external loads. The atomic and microstructural features of materials that are directly associated with the various forms of mechanical behavior are described. Using continuum models, the quantitative aspects of mechanical response are also fully examined. Analysis is possible by introducing conservation laws and suitable constitutive equations of material behavior. Conservation laws invoke well established physical principles of mass, momentum and energy conservation while constitutive equations are empirically obtained relationships relating responses to stimuli. An integrated picture of mechanical behavior of materials is developed by combining concepts of materials science, mechanical engineering and computational mechanics. All major manifestations of mechanical response of structural materials are examined including: linear elasticity, thermo elasticity, viscoelasticity, plasticity, creep, viscoplasticity and damage mechanics including fracture and fatigue. On successful completion, students will be ready to undertake advanced analytical work on classical and computational mechanics of structural materials.

2  Mechanical Engineering Materials

Mechanical Engineering systems and structures are composed of various types of materials. Reliability and life of such structures depends on the internal microstructural characteristics of the materials employed as well as on the structural design parameters. This course investigates the fundamental microstructural characteristics of engineering materials and the principles of structural mechanics employed in the design of mechanical engineering systems.

3  Structure of Materials

Ultimately, all matter is made of atomic size particles held together by interatomic forces. The mechanical response of matter to external loads is directly related to atomic cohesion.
However, the specific, quantitative form of the relationship is complicated and has yet to be elucidated.

The disciplines of metallurgy and science of materials have achieved substantial progress in our understanding of the microstructural characteristics of materials and reasonably clear pictures of the atomic constitution of most engineering materials are now available.

3.1 Classification of Materials

Engineering Materials can be classified according to various criteria. If one considers the nature of atomic arrangements in the material, two main groups emerge: crystalline materials and amorphous materials. Crystalline materials are characterized by atomic arrangements of great regularity with long range order. Atoms occupy well defined positions in a geometrically regular crystal lattice characterized by a high degree of pattern repetitiveness. In contrast, atoms in amorphous materials are located on rather more random locations and exhibit a lack of long range order. Examples of crystalline materials include metals and ceramics while glass and polymers are examples of amorphous materials. However, it is possible to produce metals with amorphous structures and polymers with crystalline arrangements as well as materials with various degrees of crystallinity.

3.2 Crystal Structures

A crystal structure is a regular three dimensional pattern of reticular locations. The basic unit of repetition of the crystalline pattern is called the unit cell. The unit cells of all known crystals belong to one of the 14 Bravais space lattices.

Crystallographic directions are indicated using the vectors associated with them with the components enclosed in square brackets. Crystallographic planes are indicated by Miller indices which are produced by listing the reciprocal intersections of the plane with the coordinate axes in round parenthesis.

3.3 Metals and Ceramics

Most elements of periodic table are metals. Metals appear mostly in three structures: face centered cubic (FCC), hexagonal close packed (HCP) and body centered cubic (BCC).

Ceramics are mainly ionic compounds and exists in many different structures, the most common being zinc blende, wurzite, perovskite, fluorite, sodium chloride, cesium chloride, spinel, corundum and crystobalite.

3.4 Polymers, Glasses, Composite and Porous Materials

Polymers are constituted by assemblies of large chain molecules with a distribution of molecular weights. Organic polymers are based on hydrocarbon chains. The chains can be linear,
branched, cross linked or ladder type. Also, depending on the pattern along the chain one has homopolymers and copolymers. Organic polymers can also be classified according to their response to heat treatment as thermoplastics or thermosets. Further, by alignment of polymer chains polymers with various degrees of crystallinity can be produced.

Glasses are materials lacking long range order. Their structure rather resembles that of a liquid. A common method for the production of glasses is by cooling from the melt while preventing crystallization.

Composite materials consists of dispersions of multiple phases in intimate contact. Most engineering materials are composites but the term is commonly used to refer to fiber reinforced materials. The properties of composites depend in a complex manner on the characteristics of the constituent phases and their interfaces.

Many structural materials have porous or cellular structures. Wood, bone and space shuttle tiles are good examples. The mechanical properties of these materials are complex functions of pore structure and characteristics.

4 The Theoretical Strength of a Crystal

Orowan first proposed a model to estimate the intrinsic mechanical strength of a crystal. He envisioned the crystal loaded in tension and failing at a certain critical load along a single crystal plane. Using a simple approximation for the cohesive strength of interatomic bonds he derived the following expression for the maximum theoretical strength

\[ \sigma_{\text{max}} \approx \frac{E}{\pi} \]

where \( E \) is the elastic modulus of the material.

A similar simple model was produced to estimate the maximum strength under shear loading. The corresponding expression is

\[ \tau_{\text{max}} \approx \frac{G}{5} \]

where \( G \) is the shear modulus of the material. The above formulae represent truly maximum values and are approximated in practice only under extreme circumstances as all sorts of microstructural defects exert a powerful influence in the determination of the actual strength of a given piece of material.

5 Solid Mechanics of Materials

Solid mechanics was originally conceived and used as a continuum theory for the estimation of the mechanical response of materials subjected to loads. In continuum theory, the details of the atomistic structure of the material are neglected. Material response is expressed in
terms of empirically determined constitutive equations of behavior. Much progress has been achieved in structural mechanics despite this assumption. Nowadays, it is recognized that some knowledge of materials science aspects is a helpful tool for understanding the mechanical response of materials. However, because of their significant engineering usefulness, it is important to have a good understanding of classical constitutive models of material behavior.

5.1 Linear Elasticity

Linear elasticity is the material deformation behavior described by Hooke’s law which states that displacement is linearly proportional to the applied load, i.e. for a point inside a material subjected to external loads $P_1, P_2, ..., P_n$, the displacement can be expressed as

$$u = \sum_{i=1}^{n} a_i P_i$$

where the coefficients $a_i$ are independent of $P_i$. A simple picture of linear elastic behavior is that of a spring.

Linear elastic behavior is well described by Hooke’s law

$$\epsilon = \frac{\sigma}{E}$$

where $\epsilon$ is the strain, $\sigma$ is the stress and $E$ is the modulus of elasticity (Young’s modulus).

A linear elastic material returns to the undeformed state once the loads are removed and the effects of multiple load systems can be computed by simple linear superposition. Moreover, the work done by the forces is calculated by multiplying the loads by the displacements and the Maxwell and Betty reciprocity relation are valid. A linear elastic material under load accumulates elastic strain energy $U$ and one has Castigliano’s theorem

$$\frac{\partial U}{\partial P_i} = u_i$$

and the associated principle of virtual work

$$\frac{\partial U}{\partial u_i} = P_i$$

5.2 Viscoelasticity

A linear elastic solid that remembers its deformation history is called a viscoelastic material. Viscoelastic behavior can be represented by combinations of springs and dashpots (pistons that move inside a viscous fluid). While linear springs instantaneously produce deformation proportional to the load, a dashpot produces a velocity proportional to the load at each instant. If a spring and a dashpot are placed in parallel one obtains Maxwell’s viscoelastic
model. If they are arranged in series, one has Voigt’s model. Finally, a series/parallel arrangement yields Kelvin’s model.

As an example, for the Voigt’s model, the relationship among load $F$, displacement $u$ and velocity $du/dt$ is

$$F = \mu u + \eta \frac{du}{dt}$$

where $\mu$ is the spring constant and $\eta$ the viscosity, and the displacement function resulting from a unit step force is a monotonic continuous function of time.

### 5.3 Plasticity

If the load applied to a piece of metal is increased from zero, the metal first deforms elastically according to Hooke’s law but at some critical threshold load, it yields and continues deforming at stresses that are much smaller than those that would be required for continued elastic deformation. This behavior is known as plasticity. Two characteristic features of plasticity are that the material deforms instantaneously and it does not return to the undeformed state when unloaded. Another characteristic frequently found is that the material strain hardens.

From a quantitative standpoint, at least two pieces of information are required to investigate plastic behavior. One first needs a yield criterion to specify the level of stress intensity at which the material ceases behaving as a linear elastic solid and residual strain remains upon unloading. The yield criterion is typically stated by saying that plastic flow begins once the effective stress

$$\sigma_{eff} = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}} = \frac{3}{2} \sigma^\prime_{ij} \sigma^\prime_{ij}^{1/2}$$

first exceeds a critical value $\sigma_0$, obtained from a simple uni-axial tensile test of a standardized specimen.

The second item required is a flow rule to describe the specific strain response of the material to the applied stress. One commonly used empirical form of the stress-strain relationship used to describe plastic flow is the so-called power law

$$\sigma_{eff} = K \varepsilon_{eff}^n$$

where $\varepsilon_{eff}$ is the effective strain and the material parameters $K$ and $n$ must also be determined from a standard tensile test.

### 5.4 Creep/Viscoplasticity

Metals at high temperatures exhibit time dependent deformation under constant load. Even very small loads may produce deformation and the body remains deformed after unloading. This behavior is known as creep or viscoplasticity.
Creep and viscoplasticity account for the combined effects of temperature, time and stress on the deformation response. A widely used, empirical relationship for the representation of the stress-strain relationship under creep/viscoelastic deformation is the Bailey-Norton law, sometimes called power law creep law. This is given by

$$\epsilon_{\text{eff}} = A\sigma_{\text{eff}}^n t^m$$

where \(\epsilon_{\text{eff}}\) is the effective creep strain, \(\sigma_{\text{eff}}\) is the effective stress, \(t\) is time, \(A\) is a temperature dependent material parameter, as are also \(n\) and \(m\), all of which must be determined from standardized creep tests.

### 5.5 Damage Mechanics, Fracture and Fatigue

Material damage is the gradual process of mechanical deterioration that ultimately results in component failure. Fracture, fatigue and creep rupture are all instances of material damage. Damage mechanics is the study of material damage based on the introduction of damage variables and their evolution. Fracture Mechanics and Fatigue Mechanics provide guidance towards understanding and controlling catastrophic structural breakdown of materials.

Fracture studies start by considering the stress concentration factor \(S\), defined as

$$S = \frac{\sigma_{\text{max}}}{\sigma}$$

where \(\sigma_{\text{max}}\) and \(\sigma\) are, respectively, the maximum (localized) and nominal stresses in the structural component. Fracture criteria specify the critical condition in a component that leads to rapid and catastrophic crack growth and propagation. Following the original idea of Griffith that assumed that all engineering materials always contain pre-existing cracks, Irwin and Orowan produced the following expression for the critical applied stress \(\sigma_c\) required for structural breakdown

$$\sigma_c = \sqrt{\frac{2EG}{\pi a(1-\nu^2)}}$$

where \(E\) is the elastic modulus, \(\nu\) is Poisson’s ratio, \(a\) is the original crack size and \(G\) is the strain energy release rate and is a physical quantity associated with the energy involved in plastic deformation at the crack tip and with the energy required to form new free surfaces at that same location.

Fatigue studies usually involve analysis of the number of loading cycles at a given stress level that are required for rapid and catastrophic fatigue crack growth and propagation. Paris et al, first proposed the following empirical relationship between the crack growth rate per cycle and the stress intensity factor range \(\Delta K = K_{\text{max}} - K_{\text{min}}\),

$$\frac{da}{dN} = C(\Delta K)^m$$
Here $C$ and $m$ are material parameters that are determined from standardized tests.

Damage mechanics aims to quantitatively represent the accrual of mechanical deterioration of a material or component subjected to certain loading. This is done by introducing a damage variable. By considering a small, representative volume element of material, with cross sectional area $dS$, the damage is defined as

$$D = \frac{dS_D}{dS}$$

where $dS_D$ be the amount of area inside an area $dS$ that is occupied by material discontinuities characterizing damage such as cracks or voids. The damage variable $D$ is a number $\in [0, 1]$. The value $D = 0$ describes undamaged material and $D = 1$ represents the ruptured component. Damage mechanics studies then focus on the quantitative determination of the response of the damage variable to specific loading conditions.

## 6 Thermodynamics of Materials

Thermodynamic principles lead to the formal statement of energy conservation and they also place restrictions on the constitutive behavior of materials.

### 6.1 The First Law of Thermodynamics

The first law of thermodynamics states that the increase of internal energy of a material system is equal to the amount of heat absorbed by it minus the amount of work done, i.e.

$$\Delta U = Q - W$$

when the first law is expressed in rate form it becomes the principle of energy conservation.

### 6.2 The Second Law of Thermodynamics

The second law of thermodynamics requires the concepts of absolute temperature and entropy. Absolute temperature is an intensive quantity and a positive number associated with the notion of hotness. Entropy is an extensive property of the system that changes as a result of interaction with the external environment and also as a result of internal processes in the system. The second law states that the change in entropy resulting from internal processes is never negative, i.e.

$$dS_i \geq 0$$
7 Continuum Thermomechanics of Materials

Continuum thermomechanics investigates the deformation behavior of materials under mechanical and thermal loads. The discipline is founded on well established conservation principles of universal applicability. The conservation principles used in continuum thermomechanics are:

- Principle of Mass Conservation (Equation of Continuity)
- Principle of Conservation of Linear Momentum (Equation of Motion)
- Principle of Conservation of Angular Momentum (Equation of Moment of Momentum)
- Principle of Conservation of Energy (Energy Equation)
- Principle of Entropy Production (Clausius-Duhem Equation)

Ultimate predictions of material deformation require incorporating constitutive equations into the formulation. While the conservation principles apply to any material, constitutive relations specify individual material responses.