One-Dimensional Dynamic Compressive Behavior of EPDM Rubber

Dynamic compressive stress-strain curves at various strain rates of an Ethylene-Propylene-Diene Monomer Copolymer (EPDM) rubber have been determined with a modified split Hopkinson pressure bar (SHPB). The use of a pulse-shaping technique ensures that the specimen deforms at a nearly constant strain rate under dynamically equilibrated stress. The validity of the experiments was monitored by a high-speed digital camera for specimen edge deformation, and by piezoelectric force transducers for dynamic stress equilibrium. The resulting dynamic stress-strain curves for the EPDM indicate that the material is sensitive to strain rates and that the strain-rate sensitivity depends on the value of strain. Based on a strain energy function theory, a one-dimensional dynamic constitutive equation for this rubber was modified to describe the high strain-rate experimental results within the ranges of strain and strain rates presented in this paper. [DOI: 10.1115/1.1584492]

Introduction

Low-impedance materials, such as rubbers, have been widely used as shock absorbers in automotive, aerospace, and portable electronics applications. Accurate material models that describe the dynamic responses of such soft materials, deforming nonlinearly at very large strains, under impact loading conditions are necessary for design optimization [1]. Furthermore, all material models need reliable experimental data in order to determine the material constants and to check the accuracy of the models over the ranges of their applications. Therefore, the dynamic properties of soft materials at various strain rates must be accurately determined under valid experimental conditions using properly designed experiments to provide physical insights for material model development and to determine constants in the models.

Most dynamic experimental techniques for high-rate properties of engineering materials are not adequate to determine the dynamic mechanical responses of rubbers at large strains and high strain rates. Rotating eccentric test machines can create relatively large strains in the specimens but only at relatively low frequencies [2]. Vibrating machines can often develop sinusoidal or other waveform input at very high frequencies but cannot achieve the large strains in the specimens needed for shock applications [3]. The dynamic destructive Charpy test cannot accurately provide a complete stress-strain history, which is necessary for developing an accurate material model for design and numerical simulation purposes. The split Hopkinson pressure bar (SHPB), originally developed by Kolsky [4], has been widely used and modified to determine the dynamic properties of a variety of engineering materials, such as metals [5], shape-memory alloys [6], ceramics [7–9], and composites [10]. SHPB experiments provide families of complete dynamic stress-strain curves as a function of strain rates, based on which dynamic material models can be developed [11,12]. Therefore, the SHPB should be an ideal tool to provide physical insights for dynamic material model development and for constant determination in the models. However, if the specimen in a SHPB is a soft material, such as an elastomer, the applicability of the conventional SHPB technique needs to be examined carefully before reliable dynamic experimental data can be produced. This applicability check is necessary because the transmitted signal may be too weak to be measured due to the drastic mismatch of impedance between the specimen and the metal bars [13,14]. The effects of specimen thickness also need to be thoroughly investigated because stress waves have very low velocities in the soft materials and may attenuate significantly when traveling through thick specimens. Because of these limitations, recent investigations have been concentrated on the modification of the conventional SHPB technique for conducting valid dynamic experiments on soft materials [13–15]. The most important modification is the re-introduction of pulse shaping technique for soft materials, as introduced in detail in Ref. [15].

In order to resolve the weak signal problem, viscoelastic bars have been used to obtain a transmitted signal with a sufficiently high signal-to-noise ratio due to the low impedance mismatch between the specimen and the bars [16–19]. However, stress homogeneity in specimen is related to bar/specimen impedance mismatch and to the rise time of the input signal [20]. A low impedance mismatch will delay the dynamic equilibrium process in the specimen, which is evident when a detailed elastic analysis of the specimen equilibrium process and the effects of bar impedance is conducted [21]. Ideally, the soft specimen should be placed in between two rigid walls so the stress level in the specimen can build up more quickly due to stress waves reflecting back and forth between the walls. This fact makes low-impedance bars undesirable. The use of viscoelastic bars also faces problems with dispersion and attenuation corrections in data reduction [19,22–26] and the dependence of the bar material on temperature, moisture level, and aging factors, which bring uncertainties into the data obtained from viscoelastic bar experiments. In order to avoid such uncertainties, metal bars with high sensitivities have been developed and validated [13,14]. We used such simpler but more suitable metal bars in this study. However, these high-sensitivity bars only satisfy the needs of sensitive instruments. Proper dynamic testing conditions must be satisfied in order to obtain valid data, which requires further modifications on the SHPB for soft material testing.

Kolsky [4], who studied the effects of thickness on the dynamic compressive stress-strain behavior of polythene, pointed out that a thick soft specimen would invalidate the assumption that the axial stresses on both sides of the specimen were nearly equal. Dioh et al. [27] pointed out that specimen thickness was an important parameter in glassy polymer tests with a SHPB. Gray et al. [28,29] performed SHPB experiments on Adiprene L-100 rubber and showed that dynamic equilibrium was achieved only during
the later stages of the experiments. Recently, a high-speed digital Imacon camera was used to monitor the dynamic deformation of RTV 630 silicon rubber \(^1\) during a conventional SHPB experiment, which showed that the specimen never deformed uniformly over the entire duration of the experiment \([15]\). Therefore, when using a SHPB to test soft materials, valid testing conditions (dynamic stress equilibrium and homogeneous deformation at a constant strain rate) during the experiment are not satisfied automatically. To obtain valid results, a reduced specimen thickness is necessary, and the shape of the loading pulse profile must be carefully controlled \([15,30]\).

Valid results from carefully designed and conducted SHPB experiments form the physical basis for dynamic constitutive model development. Due to the long history of the application of rubber materials in industry, there are numerous constitutive models based on the strain-energy function theory for rubbers under quasi-static loading conditions \([31–34]\). However, research efforts on dynamic material models have been scarce. Bergstrom and Boyce \([35,36]\) used two interacting macromolecular networks to represent the material responses of elastomers. The first network, called the equilibrium network, is represented by the Arruda-Boyce 8-chain model of rubber elasticity, and the second network is represented by another 8-chain network with a relaxed configuration. This constitutive model is applicable to a wide strain-rate range in principle, as long as the assumptions on the mechanisms (8-chain networks) are valid. However, there are a large number of constants to be determined by experiments. Empirical models such as a modified Johnson-Cook model and viscoelastic/viscoplastic constitutive models have also been used to describe the mechanical properties of polymers at various strain rates because of their simplicity \([11,37]\). However, empirical models lack the support of physical mechanisms. Wang and Yang \([38]\) developed a nonlinear viscoelastic constitutive equation based on the assumptions of nonlinear elasticity and linear viscoelasticity. Their basic formulation is based on stress relaxation functions with two different relaxation times. However, this model can only describe the dynamic properties of solid polymers at small strains, but cannot be used to describe the nonlinear behavior at large strains. Similarly, based on the strain energy function and stress relaxation function with only one relaxation time, Yang et al. \([39]\) presented a visco-hyperelastic constitutive model that combines static hyperelastic behavior and the viscoelastic constitutive model for incompressible rubber-like materials. This model provides a good description of the material behavior for the rubber only at large strains \((\lambda > 2)\). Due to the lack of proper dynamic material models, quasi-static models have been used even in dynamic simulation codes, such as DYNA3D \([40]\), to describe rubber responses under impact conditions. This indicates the strong need for accurate dynamic material models for soft materials such as rubbers.

The approach of developing constitutive models based on the strain-energy function theory has been widely accepted in recent years under quasi-static loading conditions. With the assumption of incompressible volume to simplify the constitutive model, the stress-strain relationship for a rubber under one-dimensional stress can be formulated as follows \([32]\)

\[\sigma = 2 \left( \lambda^2 - \frac{1}{\lambda} \right) \cdot \left( \frac{\partial U}{\partial I_1} + \frac{1}{\lambda} \frac{\partial U}{\partial I_2} \right) \]  

(1)

where \(\lambda\) is the stretch ratio; \(I_1\) and \(I_2\) are strain invariants,

\[I_1 = \lambda^2 + \frac{2}{\lambda} \]  

(2)

\[I_2 = \frac{1}{\lambda^2} + 2 \lambda \]  

(3)

Additionally, the strain-energy function was first derived by Mooney on the assumption that a linear stress-strain relationship existed in shear, and has been accepted by many researchers \([33,34]\). Equation (5) indicates that \(\partial U / \partial I_1 + (1/\lambda) \partial U / \partial I_2\) is a linear function of \(1/\lambda\). However, this linear function is not suitable to be applied to the rubber material that behaves nonlinearly at large strains. Furthermore, it is well known that rubbers are strain-rate sensitive. Strain-rate effects must be accounted for in the constitutive equations for rubbers to accurately describe their mechanical responses at large strains and high strain rates.

In this research, a SHPB setup modified for low-impedance material testing was employed to study the dynamic properties of Ethylene-Propylene-Diene Monomer Copolymer rubber (EPDM)\(^2\) over wide ranges of strains and strain rates. Dynamic compressive stress-strain curves were obtained under valid experimental conditions. Based on the experimental results, a one-dimensional dynamic constitutive equation has been developed that is based on a strain-energy function with strain-rate effects.

**Dynamic Experiments**

A modified SHPB setup for low-impedance material testing was used to conduct the dynamic compressive experiments on this rubber. Figure 1 shows a schematic of the experimental setup \([15]\), which is similar to a conventional aluminum SHPB except for the piezoelectric force transducers near the specimen and the pulse shaper between the striker and the incident bars. The piezoelectric force transducers were used to monitor the dynamic force equilibrium in the specimen. In a SHPB test, dynamic stress equilibrium may be checked by comparing the transmitted signal (1 wave) and the difference between the incident and reflected signals (2 waves), popularly referred to as the 1-wave, 2-wave method \([22,29,41]\). However, when the specimen is a very soft material, nearly all the incident pulse is reflected back. It is very inaccurate to compare the difference between two large-amplitude pulses (incident and reflected) and a small-amplitude pulse (transmitted). Therefore circular piezoelectric force transducers were used to directly determine the axial forces on the specimen end surfaces \([14,15,42]\). The strain gage signals were recorded using a Tektronix TDS 420A digital storage oscilloscope through ADA400A differential amplifiers. The piezoelectric transducer signals were recorded using the same oscilloscope through Kistler 5010B charge amplifiers. The pulse shapers (C11000 copper disks with various thickness and diameter) were used to precisely control the profile of the loading (incident) pulse such that the specimen deformed at a nearly constant strain rate under dynamically equilibrated stress \([30,43]\).

A specimen thickness of 1.6 mm was used to facilitate early dynamic equilibrium in the specimen during the SHPB experiments \([15,30]\). Because long cylinders of rubber will significantly attenuate the wave passing through it just as shock absorbers are designed to be, it is not possible to directly obtain material properties from experiments with such a specimen design. A thin specimen, on the other hand, can facilitate the experimental measurement of the dynamic properties, but may induce strain inhomogeneity. To closely approximate uniform deformation in the specimen, petroleum jelly was used to carefully lubricate the in-

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\(^1\)GE Silicone, 260 Hudson River Road, Waterford, NY 12188.

\(^2\)Gasket Specialties Incorporated, Emeryville, CA 94608.
interfaces between the specimen and the bars to minimize the interface friction. To evaluate the end friction effects, we also conducted experiments on different thickness specimens but at the same strain rate. The resulting stress-strain curves nearly overlapped each other, indicating that the interface friction effects were minimal after careful lubrication. In the radial directions, the choice of specimen diameter was restricted by the desired maximum strain. Rubbers are generally considered as incompressible materials during deformation. When the axial compressive strain is large enough, the cross section of the rubber specimen may exceed that of the bars, which marks the end of an experiment. For example, a specimen with an initial diameter of 12.7 mm loaded with 19 mm diameter bars will generate valid data up to an axial engineering strain of ~55%. The assumption of incompressibility leads to constant volume in the deforming rubber specimen, i.e.,

\[ \frac{\pi}{4} d_0^2 l_0 = \frac{\pi}{4} d^2 l \]

where \(d_0\) and \(l_0\) are the original specimen diameter and thickness, respectively; \(d\) and \(l\) are the current diameter and thickness respectively of the rubber specimen during deformation; and in the case of compression experiments,

\[ \frac{l}{l_0} = 1 - e_E \]

where \(e_E\) is the engineering strain of the specimen (positive in compression). The maximum allowable specimen diameter during deformation is that of the bars in order to guarantee that the rubber specimen stays within the bar end faces. The corresponding maximum original specimen diameter, \(d_0\), for a desired specimen strain, \(e_E\), and a given bar diameter, \(d_{bar}\), can thus be calculated as

\[ d_0 = d_{bar} \sqrt{1 - e_E} \]

For example, if the desired largest engineering strain is 0.7 when tested with 19.05 mm diameter bars, the corresponding maximum original diameter of the rubber specimen should be less than 10.43 mm.

In the research reported in this paper, the rubber specimens with 8.00 mm in diameter and 1.60 mm in thickness were used to study the dynamic properties of the rubber at large strains and various strain rates.

The combination of a thin specimen and a proper pulse shaper ensures that the specimen undergoes dynamic deformation under valid testing conditions. This is verified by monitoring the dynamic deformation process at the specimen edge with a high-speed digital camera (Imacon 468) and the dynamic axial force equilibrium process with piezoelectric force transducers. Figure 2 shows the high-speed backlight images of an EPDM rubber specimen edge (12.70 mm diameter, 1.60 mm thick) during dynamic compression in the modified SHPB. The apparent concave edge is actually the lubricant on the specimen/bar interfaces that have been squeezed out by the laterally expanding specimen. The one-to-one correspondence of the images with the dynamic stress-strain curve of the EPDM is also shown in Fig. 2. The fact that the specimen edge in the frames in Fig. 2 shows a nearly uniform deformation verifies that the specimen may have reached homogeneous deformation. Furthermore, engineering strains at various imaging instants can be estimated from the high-speed images. A comparison between the engineering strain estimated from the images and that integrated from the reflected signal is listed in Table 1, which displays a close agreement. When the engineering strain is larger than ~0.6, the cross-sectional area of the rubber specimen exceeds that of the bars. The true stress should stop increasing any further. This observation is in a good agreement with the results of the stress-strain curves shown in Fig. 2. The results on the dynamic equilibrium monitoring will be presented later in this paper.

Figure 3 shows a typical set of incident, reflected, and transmitted signals recorded by the digital oscilloscope from a pulse-shaped SHPB experiment on the EPDM rubber. An inspection of the pulses indicates that the shape of the incident pulse is very different from that of a conventional SHPB experiment. This is the result of controlled pulse shaping, and it is necessary to ensure the specimen deforms at a nearly constant strain rate, as indicated by the reflected signal (Fig. 3), and under dynamic equilibrium. In Fig. 3, the incident loading pulse duration is approximately from 0 mV at ~25 \(\mu\)s to ~24 mV at 175 \(\mu\)s, and that of the reflected pulse is from 0 mV at 450 \(\mu\)s to ~21 mV at 625 \(\mu\)s. The apparent length of incident pulse is 25 \(\mu\)s longer than the reflected one because the reflected pulse decreased after 625 \(\mu\)s due to the rapidly increasing transmitted pulse caused by a very thin rubber sample from excessive deformation. The corresponding transmitted pulse begins from 255 \(\mu\)s instead of the apparently 350 \(\mu\)s due to the initially weak transmitted signal. It is also noticed that there is only a difference of at most 2 mV between the incident and reflected pulses. Large error is naturally expected when 2-wave is used to compare with the initially very small 1-wave (transmitted signal) to check dynamic stress equilibrium at both ends of specimen. Therefore, piezoelectric force transducers were used to directly monitor the stress equilibrium during dynamic loading. In Fig. 4, the plateau-like region corresponds to specimen strain rates from 4600/s to 4800/s, which is considered as nearly constant strain rate after engineering strain of 10%. Figure 5 shows the comparison between the axial forces at the front-end (facing incident bar) and the back-end (facing the transmission bar) of the EPDM specimen as measured by the piezoelectric force transduc-
ers mounted near the specimen (Fig. 1). The nearly overlapping force histories indicate that the specimen deforms under dynamically equilibrated stress over the entire duration of the SHPB experiment, due to careful control of the loading pulse through pulse shaping. During the experiment, homogeneous deformation was achieved, as indicated at the specimen edge by the high-speed digital images in Fig. 2. The deformation process was at a nearly constant strain rate, as shown by the detailed strain-rate history with increasing engineering strain in Fig. 4. Furthermore, this constant-rate deformation is induced by a dynamically equilibrated applied loading, as shown in Fig. 5. Therefore, it is concluded that the resultant dynamic compressive stress-strain curve from such a valid experiment provides an accurate and reliable description of the dynamic mechanical response of the EPDM material along the direction of loading.

Following the same procedure, dynamic SHPB experiments were conducted on the EPDM rubber in the strain-rate range of \( 6.5 \times 10^2 \) to \( 4.7 \times 10^3 \) /s. The validity check was performed on each experiment. The resultant dynamic stress-strain curves at various strain rates, both in engineering and true stress-strain measures, are shown in Figs. 6 and 7, respectively. Due to the drastic amplitude differences between the lower strain-rate curves and the higher strain-rate ones, the stress-strain curves are grouped into two graphs in both Figs. 6 and 7. The results in Figs. 6 and 7 indicate that the dynamic stress-strain behavior of the EPDM rubber is highly nonlinear. The specimen responses harder and harder

<table>
<thead>
<tr>
<th>No.</th>
<th>Time (( \mu)s)</th>
<th>( L_0 ) (mm)</th>
<th>( L ) (mm)</th>
<th>Engineering strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>44.5</td>
<td>0.064</td>
<td>1.49</td>
<td>0.069</td>
</tr>
<tr>
<td>3</td>
<td>74.5</td>
<td>0.168</td>
<td>1.33</td>
<td>0.169</td>
</tr>
<tr>
<td>4</td>
<td>104.5</td>
<td>0.291</td>
<td>1.12</td>
<td>0.300</td>
</tr>
<tr>
<td>5</td>
<td>134.5</td>
<td>0.418</td>
<td>0.93</td>
<td>0.419</td>
</tr>
</tbody>
</table>

Table 1 Comparison of engineering strains integrated from reflected signal and measured from high-speed images during dynamic deformation of the EPDM rubber
to compressive stress with increasing strain as the slope of the stress-strain curve increases significantly at large strains. Furthermore, the results also indicate that the stress-strain behavior of the EPDM rubber is highly sensitive to strain rates. For example, at the true strain of 0.1, the true stress increases from ~1.0 MPa to ~7.0 MPa as the strain rate increases from 6.5 × 10^3 to 4.7 × 10^3/s. At the true strain of 0.7, the true stress increases to ~26.0 MPa at a strain rate of 4.7 × 10^3/s from ~15.0 MPa at a strain rate of 3.2 × 10^3/s. Under dynamic loading, these results indicate that material is sensitive to strain rates and that the strain-rate sensitivity depends on the value of strain. The strain-rate sensitivity at large strains (say, beyond 35% of the engineering strain) is quite different from that at small strains. Moreover, the nonlinear behavior at large strains is also different from that at small strains, as observed from Figs. 6 and 7. These characteristics caused significant difficulties in the development of the dynamic constitutive model for the EPDM rubber. A nonlinear function that couples strain and strain rate is necessary to describe the strain rate sensitivity of this rubber material.

Constitutive Equation of EPDM Rubber With Strain Rate Effects

A strain-energy function approach was adapted to accurately describe the experimental results on the dynamic compressive stress-strain behavior of the EPDM rubber. The strain-energy function \( U \) of an isotropic incompressible solid undergoing a pure homogeneous deformation can be expressed in the form of a polynomial function of strain invariants, as shown in Eq. (2), where \( C_{00} \) has been set to zero, i.e., the strain-energy function involves only nonzero powers of \( (I_1 - 3) \) and \( (I_2 - 3) \). Due to the complicated mechanical response as presented in Figs. 6 and 7, a number of strain-energy functions were tried to best describe the response while maintaining the simplicity of the model. The following strain-energy function was found to describe the dynamic stress-strain behavior for the EPDM rubber well,

\[
U = C_1(I_1 - 3) + C_2(I_2 - 3) + C_3(I_2 - 3)^2
\]

where \( C_1, C_2, \) and \( C_3 \) are constant polynomial coefficients of various powers of \( (I_1 - 3) \) and \( (I_2 - 3) \). In order to apply Eq. (9) to our uniaxial loading experiments, the stretch ratios for an incompressible material under axially symmetric deformation are expressed as

\[
\lambda_1 = \lambda, \quad \lambda_2 = \lambda_3 = \lambda^{-1/2}
\]

where \( \lambda \) is the stretch in the loading direction, and

\[
\lambda = 1 - \varepsilon_E
\]

where \( \varepsilon_E \) is the engineering strain for uniaxial compression. The constitutive relation derived from a strain-energy function for uniaxial loading can be expressed as

\[
\sigma_{\text{true}} = 2 \lambda^2 \left[ \frac{1}{\lambda} \left( \frac{1}{\lambda} \partial U \right) \frac{1}{\lambda} \partial U \right]
\]

where \( \sigma_{\text{true}} \) is the true stress in the material. The relationship between true stress and the engineering strain via the stretch (\( \lambda \)) for the EPDM rubber is represented by Eq. (1). So the experimental engineering stress-strain curves should be transformed to the cor-

Fig. 4 Strain-rate history during dynamic compression of the EPDM rubber

Fig. 5 A comparison of the axial forces on the front- and back-end of a rubber specimen

Fig. 6 Engineering stress-strain curves of the EPDM rubber at various strain rates
responding true stress-engineering strain curves, which will be fitted by the constitutive equation. Substitution of Eq. (9) into (1) leads to

\[
\sigma_{\text{sef}} = 2 \left( \lambda^2 - \frac{1}{\lambda} \right) \left( A_1 + A_2 \frac{1}{\lambda} + 2 A_3 \frac{1}{\lambda^2} \right) \tag{12}
\]

where \( A_1, A_2, \) and \( A_3 \) are combinations of \( C_1, C_2, \) and \( C_3, \) (the numerical value of each is listed in Table 2 for the EPDM rubber),

\[
A_1 = C_1 + 4C_3 \tag{13a}
\]

\[
A_2 = C_2 - 6C_3 \tag{13b}
\]

\[
A_3 = C_3 \tag{13c}
\]

Taking \( \dot{\varepsilon} = 4.7 \times 10^3 /s \) as a reference strain rate, a comparison of the dynamic true stress-engineering strain curves as obtained from the experiment and from the model is shown in Fig. 8(a). Obviously, despite an extensive effort on the constant determination, the constitutive equation, which is directly borrowed from quasi-static rubber elasticity and described by Eq. (12), does not give a good description of the dynamic material behavior at small strains \( \varepsilon < 0.15 \). However, this equation does have the potential to give a good description of the nonlinear behavior at large strains. Therefore, additional efforts to improve the model’s capability to describe the dynamic response of the EPDM rubber, especially at small strains, are necessary. The following modified constitutive equation at a reference strain rate of \( 4.7 \times 10^3 /s \) was found to describe the dynamic behavior at both small and large strains more accurately. A graphic comparison of model description and experimental data is shown in Fig. 8(b),

\[
\sigma = \sigma_{\text{sef}} \left[ 1 + D_2 \left( \frac{1}{\lambda} \right)^{-D_1} \right] \tag{14}
\]

where \( D_1 \) and \( D_2 \) are constants whose numerical values are listed in Table 2 for the EPDM rubber, and \( \sigma_{\text{sef}} \) is the true stress which was derived from Eq. (9). The behavior of the function in the bracket in Eq. (14) is that, when a large strain is applied, the term \( D_2 (1/\lambda)^{-D_1} \) becomes so small that it can be neglected. However, when a small strain is applied, \( D_2 (1/\lambda)^{-D_1} \) cannot be neglected and modifies the constitutive equation from the basic strain energy function at small strains for better accuracy.

As mentioned earlier, the strain-rate sensitivity non-linearly depends on the strain. The model described by Eq. (14) describes the dynamic EPDM rubber behavior well at only one strain rate. The

Table 2 Material constants for the constitutive model described by Eq. (16)

| \( A_1 \) | 1.1319 MPa |
| \( A_2 \) | -4.8947 MPa |
| \( A_3 \) | 0.0771 MPa |
| \( C_1 \) | 20.7107 MPa |
| \( C_2 \) | -4.4321 MPa |
| \( C_3 \) | 0.0771 MPa |
| \( B \) | -5.1962 |
| \( D_1 \) | 22.5000 |
| \( D_2 \) | 6.5075 |
| \( D_3 \) | -9.5338 |
| \( \dot{\varepsilon}_0 \) | 4700/s |

Fig. 7 True stress-strain curves of the EPDM rubber at various strain rates

Fig. 8 Dynamic stress-strain curves of the EPDM rubber at a reference strain rate of \( 4.7 \times 10^3 /s \) as described by models and the corresponding experimental results: (a) model by Eq. (12); (b) model by Eq. (14).
strain rate effects still need to be considered. The following model was therefore developed to accurately describe the strain-rate effects on the dynamic properties of the EPDM rubber at both small and large strains over the strain rate range presented in this paper,

\[ \sigma = \sigma_0 \left[ \left( \frac{1}{\lambda} \right)^{-B \lg \varepsilon_0} + \left( \frac{1}{\lambda} \right)^{-D_1} \frac{D_2}{1 + D_3 \frac{\varepsilon}{\varepsilon_0}} \right] \]  
\[ (15) \]

where \( \dot{\varepsilon}_0 \) is the reference strain rate, which is equal to 4.7 \( \times 10^3 \) s\(^{-1}\); and \( B \) and \( D_3 \) are material constants, the numerical values of which are listed in Table 2 for EPDM.

Combining Eqs. (12) and (15), the dynamic constitutive equation of the EPDM rubber including strain rate and large deformation effects can be expressed as

\[ \sigma = 2 \left( \lambda^2 - \frac{1}{\lambda} \right) \left[ A_1 + A_2 \frac{1}{\lambda} + 2 A_3 \frac{1}{\lambda^2} \right] \left[ \left( \frac{1}{\lambda} \right)^{-B \lg \varepsilon_0} + \left( \frac{1}{\lambda} \right)^{-D_1} \frac{D_2}{1 + D_3 \frac{\varepsilon}{\varepsilon_0}} \right] \]  
\[ (16) \]

Figure 9 shows a comparison between the dynamic true stress-engineering strain curves of the EPDM rubber at various strain rates as described by the model and the corresponding experimental results. The fact that the modeling and the corresponding experimental results agree well indicates that the constitutive model developed in this research accurately describes the dynamic compressive behavior of the EPDM rubber within the ranges of strains and strain rates of the experimental results presented in the paper.

Conclusions

The dynamic compressive behavior of an EPDM rubber at strain rates of 6.5 \( \times 10^3 \) s\(^{-1}\) to 4.7 \( \times 10^3 \) s\(^{-1}\) has been determined with a SHPB modified for low-impedance material testing. Pulse shaping was used to ensure that the rubber specimen deforms at a nearly constant strain rate under dynamically equilibrated stress. A high-speed digital Imacon camera was used to monitor the uniform deformation at the edges of the rubber specimens during SHPB experiments. Piezoelectric force transducers were used to record the dynamic stress equilibrium processes on the rubber specimens. These measures ensured that valid data were obtained during the SHPB experiments. Based on the experimental results, a one-dimensional dynamic constitutive model with strain-rate effects has been developed based on a strain energy function theory to accurately describe the dynamic experimental results within the ranges of strains and strain rates presented in this paper.

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References


