Predicting the Buckling Load of a Structural Tie Rod due to Axial Compression

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• **Purpose of project**: provide a designer with a sound technical guideline to predict the critical buckling load of a structural tie rod

• There are certain cases where a tie rod must be compression critical

  • This means that it must be designed to buckle at a load shortly above the design load (compression safety margin near zero)

  • In an overload event this could prevent damage to adjacent components, which otherwise could pose a hazardous condition

• Buckling test results of an actual structural tie rod are available to help validate the results of the analysis.
  • *Average experimental bucking load = 7061 lbs*
Methodology Overview

• Buckling is a material and geometric nonlinear phenomenon; in this case it must be evaluated predominantly using nonlinear FEA techniques
  • Material nonlinearity – stress / strain curve does not behave linearly at applied loading
  • Geometric nonlinearity – a structure experiences large deformations under applied loading, causing a nonlinear force/displacement relation

• First, a linear buckling analysis is performed to understand the buckling mode shape

• Second, a nonlinear analysis is conducted in three phases:
  • Nonlinear static solution and forcing incremental load steps using Modified Newton’s method with bisection convergence enhancement
  • Nonlinear static solution using arc length method
  • Nonlinear static buckling solution with eigenvalue extraction
**Modified Newton Method**

- Modified Newton method evaluates out of balance loads (difference between restoring forces and applied loads)

- A linear solution is performed using out of balance tolerance loads. If convergence fails, out of balance load is re-evaluated and stiffness matrix is updated

- Process is repeated until solution converges.

- Bisection method reduces load step in half to help solution converge

*The critical buckling load is estimated to be the applied load associated with the point after which the bisection method converges the solution after it is activated*
**Arc Length Method**

- The Arc Length method takes the Modified Newton solution and forces convergence along an arc.

- Arc length method prevents divergence at bifurcation points where the slope of the load / deflection curve becomes zero or negative
  - Modified Newton method insufficient at handling this by itself

- The node with the maximum deflection is identified and the load / deflection curve of this point is tracked post buckling

*The maximum load in the load / displacement curve of the tracked node is the estimated critical buckling load*
Nonlinear Buckling with Eigenvalue Extraction

• Nonlinear buckling analysis starts with the governing equation for eigenvalue analysis

\[
([K_0] + \lambda_{CR} \cdot [K_\sigma]) \cdot \{\varphi\} = 0
\]

• Delta stiffness matrices and displacements are evaluated at the known points of instability
  • For this reason it is critical to run the nonlinear buckling solution using load increments that cease right before the expected buckling load
  • This is why it is best to perform the NL buckling analysis last

• Two subcases with incremental loads are needed to calculate the critical load based on equation to the right

\[
\{P_{CR}\} = \{P_n\} + \lambda \cdot \{\Delta P\}
\]

\[
\{\Delta P\} = \{P_n\} - \{P_{n-1}\}
\]

• The FEA output file provides the eigenvalues.

• The critical load is based on the equation using the eigenvalue from the second load case
## Analysis / Results

<table>
<thead>
<tr>
<th>Analysis Method</th>
<th>Brief Analysis Summary</th>
<th>Critical Buckling Load</th>
<th>Error Margin</th>
</tr>
</thead>
</table>
| Modified Newton       | • 1st subcase loads tie rod up to 6000 lbs in 5 increments  
                        • 2nd subcase loads tie rod from 6000 to 7000 lbs in 5 increments  
                        • Bisection method converged at .100 load step in second subcase                                                                                                                                       | 6100 lbs               | 13.6%        |
| Arc Length            | • 1st subcase loads tie rod to 6000 lbs in 5 increments with instigative preload to induce buckling shape  
                        • 2nd subcase employs arc length method                                                                                                                                                               | 6134 lbs               | 13.1%        |
| NL Buckling           | • 1st subcase loads tie rod up to 5000 lbs in 5 increments  
                        • 2nd subcase loads tie rod from 5000 to 6000 lbs in 5 increments  
                        • 1st mode Eigenvalues for 1st & 2nd subcase resp. are .812 & .562                                                                                                                                     | 6112 lbs               | 13.4%        |
Conclusions

• All three NL analysis methods produce results with great precision

• Error margin of around 13% is considered accurate for a nonlinear buckling validation study based on existing literature in the field

• It is recommended that a designer incorporate all three NL analysis techniques if possible, in order to substantiate results

• Results with Arc Length method are considered least conservative, although only vary by a few tenths of a percentage point.
  • May be a good choice for analysis that minimizes compression margin of safety