Project Scope:
The final exam focused on looking at the deflection of a rectangular plate for 8 ply symmetric laminates with different angle orientations. Specifically problem 1 focused on a cross ply $[0/+90/-90/0]_s$ orientation while problem 3 focused on an angle ply $[0/+45/-45/0]_s$ orientation.

This project will look at a two additional orientations which are $[0/+45/-45/+90]_s$ and $[0/+45/0/+90]_s$ to determine which lay-up orientation is more resistant to deflection in the $z$ direction. Lastly a pressure load will be applied to the short side along the $x$-direction (parallel to the long side). The deflection in the $z$-direction with the applied side load will be compared to the original deflections.

The results will be calculated via Rayleigh – Ritz method using Maple and finite element method using Ansys.

Given:
Plate dimensions: 0.2m x 0.1m
Lamina thickness: 0.00025m
Pressure load (q0) in the $z$-dir: $1e6$ Pa
Pressure load in the $x$-dir: $1e6$ Pa
Material: T300/5208 Graphite-Epoxy
Properties: $E_1 = 181$ GPa, $E_2 = 10.3$ GPa, $G_{12} = 7.17$ GPa, $\nu_{12} = 0.28$, $\nu_{23} = 0.59$
Results for [0/+90/-90/0], plate

Exact solution using Maple

\[
\text{plot3d}(w, x = 0..a, y = 0..b);
\]

\[
\text{evaf}(\text{subs}(x = \frac{a}{2}, y = \frac{b}{2}, w)); \quad 0.02495483655
\]

Rayleigh – Ritz solution using Maple

\[
\text{plot3d}(w, x = 0..a, y = 0..b);
\]

\[
\text{evaf}(\text{subs}(x = \frac{a}{2}, y = \frac{b}{2}, w)); \quad 0.02496818290
\]
Finite element method using Ansys

![Finite element method using Ansys](image_url)
Results for $[0/+45/-45/0]_s$ plate

Exact solution using Maple

```maple
> plot3d(w, x=0..a, y=0..b);
```

```maple
> evalf(subs(\{x = \frac{a}{2}, y = \frac{b}{2}\}, w));
```

0.03464378201

Rayleigh–Ritz solution using Maple

```maple
> plot3d(w, x=0..a, y=0..b);
```

```maple
> evalf(subs(\{x = \frac{a}{2}, y = \frac{b}{2}\}, w));
```

0.03549123849
Finite element method using Ansys
Results for $[0/+45/-45/90]_s$ plate

**Exact solution using Maple**

```
> plot3d(w, x=0..a, y=0..b);
```

```
> evalf(subs(\{x = a/2, y = b/2\}, w));
0.03342016029
```

**Rayleigh – Ritz solution using Maple**

```
> plot3d(w, x=0..a, y=0..b);
```

```
> evalf(subs(\{x = a/2, y = b/2\}, w));
0.03417965574
```
Finite element method using Ansys
Results for $[0/+45/0/90]_s$ plate

Exact solution using Maple

```maple
plot3d(w, x = 0..a, y = 0..b);
```

```maple
evalf(subs(\{x = a/2, y = b/2\}, w));
```

Rayleigh – Ritz solution using Maple

```maple
plot3d(w, x = 0..a, y = 0..b);
```

```maple
evalf(subs(\{x = a/2, y = b/2\}, w));
```

0.03932290920

0.04271934947
Finite element method using Ansys
Deflection in Z-direction for [0/90/-90/0]_s plate with Pressure load in X-direction
Deflection in Z-direction for \([0/45/-45/0]_s\) plate with Pressure load in X-direction
Deflection in Z-direction for [0/45/-45/90]_s plate with Pressure load in X-direction
Deflection in Z-direction for [0/45/0/90]_s plate with Pressure load in X-direction
Stress in the X-direction for [0/90/-90/0]_s plate with no Pressure load in X-direction
Stress in the X-direction for $[0/45/-45/0]_4$ plate with no Pressure load in X-direction
Stress in the X-direction for \([0/45/-45/90]_s\) plate with no Pressure load in X-direction
Stress in the X-direction for [0/45/0/90]ₚ plate with no Pressure load in X-direction.
Stress in the X-direction for [0/90/-90/0]_s plate with Pressure load in X-direction
Stress in the X-direction for [0/45/-45/0]_s plate with Pressure load in X-direction
Stress in the X-direction for [0/45/-45/90], plate with Pressure load in X-direction
Stress in the X-direction for [0/45/0/90]s plate with Pressure load in X-direction

Summary of Results

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[0,+90,-90,0]s</td>
<td>[0,+45,-45,0]s</td>
<td>[0,+45,-45,90]s</td>
<td>[0,+45,0,90]s</td>
</tr>
<tr>
<td>Defl in Z-dir w/ no X-dir Pressure (in)</td>
<td>0.025033</td>
<td>0.035771</td>
<td>0.034433</td>
<td>0.04362</td>
</tr>
<tr>
<td>Defl in Z-dir w/ X-dir Pressure (in)</td>
<td>0.025033</td>
<td>0.035771</td>
<td>0.034433</td>
<td>0.04362</td>
</tr>
<tr>
<td>Stress in X-dir w/ no X-dir Pressure (Pa)</td>
<td>1.10E+09</td>
<td>1.45E+09</td>
<td>1.38E+09</td>
<td>1.87E+09</td>
</tr>
<tr>
<td>Stress in X-dir w/ X-dir Pressure (Pa)</td>
<td>2.04E+09</td>
<td>2.32E+09</td>
<td>2.68E+09</td>
<td>2.77E+09</td>
</tr>
<tr>
<td>% increase in stress w/ load application</td>
<td>85.5</td>
<td>60.0</td>
<td>94.2</td>
<td>48.1</td>
</tr>
<tr>
<td>% increase in deflection vs min</td>
<td>N/A</td>
<td>42.9</td>
<td>37.6</td>
<td>74.2</td>
</tr>
<tr>
<td>% increase in stress vs min w/ no load</td>
<td>N/A</td>
<td>31.8</td>
<td>25.5</td>
<td>70.0</td>
</tr>
<tr>
<td>% increase in stress vs min w/ load</td>
<td>N/A</td>
<td>13.7</td>
<td>31.4</td>
<td>35.8</td>
</tr>
<tr>
<td>Min deflection in Z-direction</td>
<td>0.025033 m</td>
<td>Orientation</td>
<td>[0,+90,-90,0]s</td>
<td></td>
</tr>
<tr>
<td>Min stress in X-direction w/ no load</td>
<td>1.10E+09 Pa</td>
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<td></td>
</tr>
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<td>Min stress in X-direction w/ load</td>
<td>2.04E+09 Pa</td>
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<td></td>
</tr>
</tbody>
</table>
Conclusion

There are several conclusions from this study:

1. The [0,90,-90,0]s orientation is the one that experiences the least amount of deflection in the z-direction. This is expected because the direction of the fibers are along the primary axes. Therefore as the plate deflects these fibers experience tension which is where they have the highest capability. The other orientations have fibers at an angle from the primary axes and thus some of the deflection is taken up by tension in the matrix which is weaker, thus causing more deflection.

2. The [0,90,-90,0]s orientation is the one that experiences the least amount of stress in the x-direction whether a pressure load is applied in the x-direction or not. This is somewhat unexpected because one would assume that the orientations with the 45 degree fibers such as [0,+45,-45,0]s would experience less stress in the x-direction. The reason is that for the lamina with the 45 degree fiber orientation the load is shared between the fibers and the matrix, while for the lamina with the 90 degree orientation more load is taken by the matrix. Since the matrix has a lower modulus than the fibers one would expect that it would experience a higher stress.

3. The deflection in the z-direction does not change for any of the laminates.

4. The stress in the x-direction more than doubles for all the lamina except for the [0,45,0,90]s where it increases by 48%.