Theory of creep deformation with kinematic hardening for materials with different properties in tension and compression

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Received in final revised form 5 December 2003
Available online 6 March 2004

Abstract

A constitutive model for creep deformation that describes the loading-history-dependent behavior of initially isotropic materials with different properties in tension and compression under stress vector rotations limited by 50–60° is presented within a thermodynamic framework. In the proposed constitutive model a kinematic hardening rule is adopted. This model also introduces an effective equivalent stress in the creep potential that is based on the first and second invariants of the effective stress tensor, and on the joint invariant of the effective stress tensor and eigenvector associated with the maximum principal Cauchy stress. The formulation of the kinematic hardening rule is presented and discussed. All the material parameters in the model have been obtained from a series of proposed basic experiments with constant stresses. These model parameters are then used to predict the creep deformation of the aluminum alloy under multiaxial loading with constant stresses, and under non-proportional uniaxial and non-proportional multiaxial loadings for both isothermal and nonisothermal processes.

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Keywords: Constitutive behavior; Creep; Kinematic hardening; Asymmetric effects; Thermodynamics
1. Introduction

The phenomenon of time-dependent irreversible deformation of polycrystalline materials under constant load at high temperatures is now termed “creep”. A creep test of very long duration at high temperatures for metallic materials produces significant creep strains during the application of the load, and smaller thermoelastic strains and much smaller plastic ones in comparison to the creep strains. For the restriction to small total strains under high temperature creep conditions the additive decomposition of the total strains (or rate strains) to thermoelastic and creep parts is permissible (Boyle and Spence, 1983; Gurtin, 2003; Rabotnov, 1969) neglecting the plastic ones. The basic problem here is to formulate the constitutive equation for the creep strain (or creep strain rate) (Boyle and Spence, 1983; Rabotnov, 1969; Kreml and Khan, 2003). In the following, the subject of the present paper will be only the description of the primary and secondary stages of the creep deformation. We shall not consider the tertiary stage of the creep deformation, development of creep damage, damage-induced anisotropy and creep rupture, which are discussed, for example, in Dyson and Gibbons (1987); Hayhurst (1972); Kowalewski et al. (1994); Othman et al. (1993).

The use of polycrystalline materials in structural components at elevated temperatures under non-proportional mechanical loading with changing thermal conditions for a prolonged period of time requires the studying of their creep deformation under changing multiaxial stresses. The features of creep deformation for polycrystalline materials may be investigated experimentally. Some of these creep features for initially isotropic polycrystalline metals and alloys are the following:
1. different properties in tension and compression,
2. anisotropic strain hardening.

Tension/compression asymmetry in creep behavior of materials is determined by comparing tensile and compressive creep curves obtained under uniaxial tension and uniaxial compression tests at the same temperature, and for one and the same absolute value of constant stress (Zolochevskii, 1982, 1988). In this case one has two different creep curves (one in tension, and another in compression). Different behavior in tension and compression under creep conditions was experimentally observed for many polycrystalline materials, such as light alloys (El-Shennawy et al., 1999; Gorev et al., 1978; Hostert, 1975; Khojasteh-Vahabzadeh, 1991; Lucas and Pelloux, 1981; Nikitenko et al., 1971; Rabotnov, 1969; Rubanov, 1987; Sosnin, 1970; Tsvelodub, 1991), high strength steels (Rabotnov, 1969; Rix, 1997), gray cast irons (Nechtelberger, 1985), polymers (Zolochevskii, 1988), rock salt (Chan et al., 1994), and ceramic polycrystal (Pintschovius et al., 1989; Wereszczak et al., 1999). For metallic materials the difference in the creep strain under tension and compression ranges from 2 to 10 times, and for crystalline ceramics this difference can be 289 times. The above-mentioned feature of the materials under creep conditions is similar to the well-known tension–compression asymmetry for the inelastic deformation by slip for polycrystals and single crystals (Balasubramanian and Anand, 2002; Iyer and Lissenden, 2003; Karaman et al., 2000), when the deformation re-
sistance of different classes of slip systems can be substantially different in tension and compression.

The classical theories (Boyle and Spence, 1983) of creep deformation use the assumption of isotropic strain hardening. However, various experimental data under uniaxial and multiaxial non-proportional loadings (Findley et al., 1979; Murakami and Ohno, 1982; Ohashi et al., 1982; Rabotnov, 1969) show that the creep behavior of materials is a loading-history-dependent anisotropic phenomenon. Current principal directions of the Cauchy stress tensor and of the creep strain rate tensor do not coincide under non-proportional loading.

Since the pioneering works of Bailey (1926) and Orowan (1946), quite a large number of constitutive equations for creep deformation have been proposed to describe anisotropic hardening of materials under changing multiaxial stresses. These models can be categorized as viscoelastic–viscoplastic (Chaboche, 1997; Findley and Lai, 1981; Freed and Walker, 1993; Krempf and Gleason, 1996; Lemaitre and Chaboche, 1990; Rubin and Bodner, 1995; Shapery, 1969; Yang, 1997; Haupt and Kersten, 2003; Lubarda et al., 2003), micromecanical (Gittus, 1978; Hart, 1976; Lagneborg, 1972; Miller, 1976; Mughrabi, 1975; Quesnel and Tsow, 1981; Santaoja, 1991) and phenomenological (Faruque et al., 1996; Kawai, 1995; Klebanov, 1999; Malinin and Khadjinsky, 1972; Murakami and Ohno, 1982; Ohashi et al., 1982; Ponter, 1975; Robinson, 1978; Trampczynski and Mroz, 1992; Scheidler and Wright, 2003). Most of the models under discussion are based on the introduction of kinematic hardening or a combination of isotropic and kinematic hardening to describe the anisotropic creep hardening. Authors of these models accept the tension/compression symmetry in creep behavior of materials in reference to the experiments by Ding and Findley (1984), Hayhurst (1972, 1981), Johnson et al. (1956), and Tilly and Harrison (1972).

Constitutive models of viscoplasticity for porous materials (Bammann, 1990; Cocks, 1989; Duva and Hutchinson, 1984; Mähler et al., 2001; Marin and McDoowell, 1996; Michel and Suquet, 1992; Sofronis and McMeeking, 1992; Tvergaard, 1981) have received special attention. In contrast to classical theories of creep deformation which are based on the creep incompressibility, the structure of constitutive equations for porous materials reflects volumetric inelastic deformation and hydrostatic pressure dependence. However, these constitutive models of viscoplasticity for porous materials based on a symmetric, “elliptic” potential and the assumption of same behavior in uniaxial tension and uniaxial compression cannot correctly describe the experimental data (Geindreau et al., 1999; Mosbach et al., 1997) related to the tension–compression asymmetry.

Thus, up to now, to the best of the authors’ knowledge, in the literature there is no theory of creep deformation, which is able to reproduce both the different properties of materials under tensile and compressive loading types, and their corresponding anisotropic hardening. The previous papers by Betten et al. (1998, 1999, 2003), Voyiadjis and Zolochevsky (1998a,b, 2000), Zolochevskii (1982, 1988), Zolochevsky (1991, 1995), and Zolochevsky and Obataya (2001), described the creep behavior of initially anisotropic and isotropic materials with different properties in tension and compression under the assumption of isotropic hardening. They were used as the
background for the models in the recently published papers by Kawai (2002) and Mahnken (2003) which also did not address the materials with the tension–compression asymmetry and anisotropic hardening. The aim of the present paper is to propose a new theory for creep deformation of initially isotropic materials in which the deformation-induced anisotropy related to the past history of loading as well as the different creep behavior of materials in tension and compression are taken into account for. In the following, one will consider describing the above-mentioned features of the creep response due to small creep strains within the framework of the phenomenological point of view. The theory for creep deformation will be oriented to the consideration of the kinematic hardening under stress vector rotations limited by 50–60° only (Klebanov, 1999). We will not adopt in the present paper the hypothesis of creep incompressibility. First, it was experimentally established (Nikitenko et al., 1971; Rubanov, 1987) that many light alloys with different behavior in tension and compression are mostly compressible under creep conditions. Second, as shown by Tsvelodub (1991), the assumption regarding creep compressibility is leading to the fact that the first invariant of the stress tensor cannot be included in the list of arguments in the expression for the equivalent stress in the creep potential. Therefore, in this case we have limitations for the construction of the constitutive equation for creep deformation. Third, it was shown (Betten et al., 2003) that the previous theory of the authors with isotropic creep hardening (Betten et al., 2003; Voyiadjis and Zolochevsky, 2000) for materials with different behaviors in tension and compression is capable of predicting a creep compressibility and volume change of the material under uniaxial loading as well. The present paper will not consider describing the set of phenomena occurring in polycrystalline materials under cyclic loading such as ratcheting effects and cyclic creep.

2. Constitutive model

2.1. Thermodynamic potential

Let us consider the formulation of the constitutive model for describing the phenomena of thermoelasticity and creep deformation for initially isotropic materials with kinematic creep hardening within the general thermodynamic framework (Lemaitre and Chaboche, 1990; Chaboche, 1997) based on the internal variables neglecting plasticity, viscoplasticity and its coupling with creep. The components $E_{kl}$ of the total infinitesimal strain tensor under creep conditions are assumed to be the sum of the thermoelastic components $e_{kl}$ and the creep ones $\varepsilon_{kl}$, i.e.,

$$E_{kl} = e_{kl} + \varepsilon_{kl}.$$  

In this way, the Helmholtz free specific energy $\psi$, taken as the thermodynamic potential, is also the sum of two terms: the thermoelastic part $\psi_1$ and the creep one $\psi_2$. Because the microstructure of materials cannot evolve in an unstable way with the variations in temperature $T$, the creep part $\psi_2$ does not depend on the variable $T$, ...
although the temperature may be included in $\psi_2$ as a parameter. Thus, one can write the expression for the Helmholtz free specific energy as follows:

$$\psi = \psi_1(e_{kl}, T) + \psi_2(X_{kl}),$$  \hspace{1cm} (2)$$

where $X_{kl}$ is the second-order tensorial flux variable corresponding to the back stress tensor $\rho_{kl}$. The back stress tensor is connected with such micromechanisms as dislocation movement, the appearance of new dislocations, dislocation pile-ups on obstacles, escape of dislocations from their glide planes, dislocation annihilation, etc. The flux variable $X_{kl}$ is the associated variable for $\rho_{kl}$.

The thermoelastic part of the Helmholtz free specific energy may be introduced as

$$\psi_1 = \frac{1}{\rho} \left[ \frac{1}{2} (\dot{\lambda}_0 e_1^2 + 2 \mu_0 e_2) - (3 \lambda_0 + 2 \mu_0) \lambda_0 \theta e_1 \right] - \frac{c_0}{2 T_0} \theta^2,$$  \hspace{1cm} (3)$$

where $e_1 = e_{kl} \delta_{kl}$, $e_2 = e_{kl} e_{kl}$, $\delta_{kl}$ is the Kronecker delta, $\rho$ is the material density that can be considered to be constant, $\lambda_0$ and $\mu_0$ are Lame’s coefficients, $\lambda_0$ is the coefficient of dilatation, $c_0$ is the specific heat at constant strain, $\theta = T - T_0$, and $T_0$ is a reference temperature. The temperature difference, $\theta$ is small with respect to $T_0$.

The Cauchy stress tensor $\sigma_{kl}$ may be defined as

$$\sigma_{kl} = \rho \frac{\partial \psi}{\partial e_{kl}}.$$  \hspace{1cm} (4)$$

Substituting Eqs. (2) and (3) into Eq. (4), one obtains the following equation for the classical linear thermoelasticity

$$\sigma_{kl} = \lambda_0 e_1 \delta_{kl} + 2 \mu_0 e_{kl} - (3 \lambda_0 + 2 \mu_0) \lambda_0 \theta \delta_{kl}.$$  \hspace{1cm} (5)$$

The temperature $T$ is associated with the entropy density $S$, and their relation can be defined as

$$S = \frac{\partial \psi}{\partial T}.$$  \hspace{1cm} (6)$$

Using Eqs. (2), (3) and (6), one obtains

$$S = \frac{1}{\rho} (3 \lambda_0 + 2 \mu_0) \lambda_0 e_1 + \frac{c_0}{T_0} \theta.$$  \hspace{1cm} (7)$$

The introduction of the creep part $\psi_2(X_{kl})$ in the Helmholtz free-specific energy will be given later in this section of the paper.

2.2. Creep potential

Let us incorporate the back stress tensor $\rho_{kl}$ into the creep constitutive equation, in addition to the Cauchy stress tensor and temperature. The difference between the Cauchy stress tensor and the back stress tensor is often defined as the effective stress tensor $\lambda_{kl}$, i.e.,

$$\lambda_{kl} = \sigma_{kl} - \rho_{kl}.$$  \hspace{1cm} (8)
One assumes the existence of a creep potential such that

\[ F = \dot{\varepsilon}_e^2 - \varphi^2(\dot{\varepsilon}_e) = 0. \] (9)

Eq. (9) describes a certain surface in the space of the Cauchy stress tensor components where the center of this surface is defined by the back stress tensor. The first term in Eq. (9) describes the form of this surface, and the second one defines the current position of the surface. One assumes that the subsequent surface described by Eq. (9) moves without changing its initial shape and size. Here \( \dot{\varepsilon}_e \) is the equivalent effective stress \( (\dot{\varepsilon}_e \geq 0) \) which will be defined later in this section of the paper, and \( \dot{\varepsilon}_c \) is the equivalent creep strain rate \( (\dot{\varepsilon}_c \geq 0) \) which will be considered as a magnitude which is multiplied by \( \lambda \), and yields the following mixed invariant:

\[ L = \lambda \dot{\varepsilon}_e \dot{\varepsilon}_c. \] (10)

Here \( \dot{\varepsilon}_{kl} \) is the creep infinitesimal strain rate tensor, and the dot above the symbol denotes the derivative with respect to time \( t \). It follows from the last definition that

\[ \lambda \dot{\varepsilon}_e = L. \] (11)

Eq. (11) is similar to the Hill-type definition (Hill, 1948, 1987) for the appropriate equivalent measures of stresses and non-elastic strain rates with respect to the principle of work-equivalence that is often used in different theories of plasticity, viscoplastioicity and creep (Benzeraga and Besson, 2001; Boyle and Spence, 1983; Lemaitre and Chaboche, 1990; Malinin and Khadjinsky, 1972; Rabotnov, 1969; Zolotchevsky, 1982, 1988; Zolotchevsky, 1991, 1995). Of course, Eq. (11) reflects only one way for a possible non-elasticity modeling of a class of materials. For example, some authors (Chen and Zhang, 1991; Grewolls and Kreißig, 2001) have used the constitutive equations of plasticity without making use of the Hill-type definition.

The creep strain rate tensor can now be defined as

\[ \dot{\varepsilon}_{kl} = \lambda \frac{\partial F}{\partial \sigma_{kl}} = \lambda \frac{\partial F}{\partial \sigma_{kl}} = 2\lambda \varepsilon_{e} \frac{\partial \varepsilon_{e}}{\partial \sigma_{kl}}, \] (12)

where \( \lambda \) is a scalar multiplier that will be determined later in this section.

Let \( \mathbf{n} = (n_k)_{k=1}^3 \) be an eigenvector associated with the maximum principal Cauchy stress. One can now assume that the deformation-induced anisotropy in creep deformation governed by the past history of loading, and different creep properties in tension and compression may be described using the back stress tensor, the eigenvector \( \mathbf{n} \) and the effective stress tensor. In other words, the equivalent effective stress may be considered as a function, depending on the effective stress tensor \( \sigma_{kl} \) and the eigenvector \( \mathbf{n} \). The integrity basis for the symmetric second-order tensor \( \sigma_{kl} \) and the vector \( \mathbf{n} \) consists of the following five irreducible invariants (Spencer, 1971)

\[ I_1 = \sigma_{kl}n_kn_l, \quad I_2 = \sigma_{kl}n_kn_l, \quad I_3 = \sigma_{kmn}n_kn_l, \quad I_4 = \sigma_{kl}n_kn_l, \] (13)

Obviously, in order to describe different properties of materials in tension and compression, both odd and even invariants in the list given by Eq. (13) must be used. One then introduces the effective stress intensity
\[ \alpha_i = \sqrt{\frac{3}{2}S_{kl}S_{kl}}, \]  
where \( S_{kl} \) is the effective stress deviator, i.e.,
\[ S_{kl} = \alpha_{kl} - \frac{1}{3}I_1 \delta_{kl}. \]

Note that the effective stress intensity is often used in the classical theories of creep deformation with anisotropic hardening (Kawai, 1995; Klebanov, 1999; Lemaitre and Chaboche, 1990; Malinin and Khadjinsky, 1972). It is not difficult to deduce that by using Eqs. (13)–(15), the following relationship can be obtained
\[ \alpha_i = \sqrt{\frac{3}{2}I_2 - \frac{1}{2}I_1^2} \]
between the effective stress intensity and the invariants \( I_1, I_2 \).

One can also note that in the case when tensor \( \rho_{kl} \equiv 0 \) the invariant \( \alpha_1 \) coincides with the maximum principal Cauchy stress
\[ \sigma_1 = \sigma_{kl}n_kn_l \]
and the invariant \( \alpha_i \) coincides with the Cauchy stress intensity
\[ \sigma_i = \sqrt{\frac{3}{2}\sigma_{kl}\sigma_{kl}}, \]
where
\[ \sigma_{kl} = \sigma_{kl} - \frac{1}{3}\sigma_{mn}\delta_{kl} \]
is the Cauchy stress deviator. In the classical theories of creep (Boyle and Spence, 1983; Rabotnov, 1969) that are based on the isotropic hardening of the Huber–von Mises-type materials with the same properties in tension and compression, the Cauchy stress intensity is defined as the equivalent stress, and the creep strain rate intensity
\[ \dot{\varepsilon}_i = \sqrt{\frac{3}{2}\dot{p}_{kl}\dot{p}_{kl}} \]
is defined as the equivalent creep strain rate. Here
\[ \dot{p}_{kl} = \dot{\varepsilon}_{kl} - \frac{1}{3}\dot{\varepsilon}_{mn}\delta_{kl} \]
is the creep strain rate deviator. Furthermore, the following relation for these measures of the stress and strain states is given by
\[ \sigma_i\dot{\varepsilon}_i = W, \]
where
\[ W = \sigma_{kl}\dot{\varepsilon}_{kl} \]
is the specific energy dissipation rate. Eq. (22) reflects the principle of work-equivalence proposed by Hill (1948, 1987). Note also that Eq. (11) considered as generalization of Eq. (22) was used in most of the classical theories with anisotropic creep hardening.
One now returns to the definition of the equivalent effective stress under discussion. There are different ways to introduce the expression for the equivalent effective stress using the list of the invariants given above. It is necessary to introduce such an expression which leads to the model that can distinguish between the creep behaviors under different modes of loading and can include a number of the well known in the literature creep models as particular cases. Very important feature concerning this subject is that many metallic materials have not only different properties in tension and compression, but also independent creep behavior under conditions of pure shear. Therefore, one may introduce into the expression for the equivalent effective stress three independent in a general case material parameters and three invariants from the list given above. Neglecting then the influence of the invariants $I_3$ and $I_4$ as second-order effects, one can assume the following expression for the equivalent effective stress

$$a_e = \sqrt{AI_2 + \beta BI_1^2 + \gamma C x_1},$$  \hspace{1cm} (24)

where $A$, $B$ and $C$ are some temperature-dependent material parameters, and $\beta$ and $\gamma$ are numerical coefficients which take into account the specific weight for the odd invariants in expression (24).

A number of comments need to be made in reference to expression (24) of the equivalent effective stress. Firstly, in definition (24) the equivalent effective stress is a homogeneous function of the effective stresses. The first term in Eq. (24)

$$x_0 = \sqrt{AI_2 + \beta BI_1^2}$$  \hspace{1cm} (25)

can be considered as a generalization of the effective stress intensity given by Eq. (16) to the materials with two independent material characteristics. Actually, assuming in Eq. (25) $A = \frac{1}{2}$ and $\beta B = -\frac{1}{2}$, one arrives at Eq. (16). The second term in Eq. (24) is proportional to the joint invariant of the effective stress tensor and eigenvector associated with the maximum principal Cauchy stress.

Secondly, expression (24) contains three material parameters ($A$, $\beta B$ and $\gamma C$) and three invariants including two odd invariants $I_1$ and $x_1$, and therefore it gives the opportunity to describe different properties in tension and compression as well as the independent law for creep deformation under conditions of pure torsion that it was experimentally established for many initially isotropic polycrystalline materials (Altenbach et al., 1995; Gorev et al., 1978; Rabotnov, 1969; Rubanov, 1987; Tsvelodub, 1991).

Thirdly, the coefficients $\beta$ and $\gamma$ are introduced in expression (24) for convenience only. It is impossible to find the coefficients $\beta$ and $\gamma$ separately from parameters $A$ and $B$, respectively. Obviously, definition (24) is not unique for the equivalent effective stress. In the same time, expression (24) has a general form and includes as the particular cases a number of expressions well known in the literature. For example, assuming $A = \frac{1}{2}$, $\beta B = -\frac{1}{2}$ and $\gamma = 0$, one arrives at the expression for the equivalent effective stress, $x_e = x_1$, in the creep potential for the case of the theory (Malinin and Khadjinsky, 1972) for creep deformation with kinematic hardening for the Huber–
von Mises-type materials with the same properties in tension and compression. As it is known (Zolochevskii, 1982, 1988), comparing creep curves obtained under uniaxial tension tests, uniaxial compression tests and pure torsion tests for the same temperature using specimens of the same orientation it is possible to determine a rule for the influence of the kind of loading. In the case under discussion the values of the creep strain intensity, chosen for one and the same value of time, do not depend on the kind of loading. Thus one has one single creep curve for uniaxial tension, uniaxial compression and pure torsion. If in the other case one assumes \( p_{kl} \equiv 0 \) and \( \chi_{kl} \equiv \sigma_{kl} \) in expression (24), one arrives at the equivalent stress (Betten et al., 1998):

\[
\varepsilon_e = \sqrt{A \sigma_{kl} \sigma_{kl} + \beta B \sigma_{mn} \sigma_{mn}} + \gamma C \sigma_i
\]  

(26)

for the case of isotropic hardening of materials with characteristics that are essentially sensitive to the type of loading. The last expression implies that the values of the creep strain intensity, taken from uniaxial tension tests, uniaxial compression tests and pure torsion tests at the same temperature, and chosen for one and the same constant value of the Cauchy stress intensity and for one and the same value of time, are essentially depending on the kind of loading. Thus one has three different creep curves (the first in tension, the second in compression, and the third in torsion).

Taking into account (Altenbach et al., 1995) that \( \sigma_1 = \frac{1}{3} \sigma_i \sin (\xi + \frac{2\pi}{3}) + \frac{1}{3} \sigma_{nn} \) and assuming in Eq. (26) \( A = \frac{1}{3} A_0^2 \), \( \beta B = -\frac{1}{3} A_0^2 \), one arrives at the expression \( \varepsilon_e = A_0 \sigma_i + \frac{1}{3} \gamma C \sigma_i (\sqrt{3} \cos \xi - \sin \xi + \frac{2\pi}{3}) \) for the equivalent stress based on the first invariant of the Cauchy stress, the Cauchy stress intensity and the angle of the kind of the Cauchy stress state \( \xi \) (or third invariant of the Cauchy stress deviator \( J_3 \)), \( \sin 3\xi = -\frac{\sqrt{3}}{2} \frac{J_3}{\sigma_i} \). \( J_3 = \frac{1}{3} \sigma_{kl} \sigma_{ml} \sigma_{mk} \). For the pressure-dependent processes under the condition of \( \frac{\sigma_{nn}}{\sigma_i} \gg \sqrt{3} \cos \xi - \sin \xi \) one obtains the expression \( \varepsilon_e = A_0 \sigma_i + \frac{1}{3} \gamma C \sigma_{nn} \) of the equivalent stress in the potential of the Schleicher–Drucker–Prager (Schleicher, 1926; Drucker and Prager, 1952). In the case with \( \frac{\sigma_{nn}}{\sigma_i} \ll \sqrt{3} \cos \xi - \sin \xi \) one obtains the expression \( \varepsilon_e = \sigma_1 (A_0 + \frac{\sqrt{3}}{2} \gamma C \cos \xi - \frac{1}{4} \gamma C \sin \xi) \) of the equivalent stress in the potential of the Drucker–Rabotnov-type (Drucker, 1949; Rabotnov, 1969) based only on the invariants \( \sigma_1 \) and \( \xi \) (or \( J_3 \)). Note also that formula (26) under the condition of \( \gamma = 0 \) was proposed by Green (1972), but its particular cases under the condition \( A = \frac{1}{2} A_0^2 \), \( \beta B = -\frac{1}{2} A_0^2 \) were discussed earlier by Sdobyrev (1959) with \( A_0 = \gamma C = \frac{1}{2} \) and by Rabotnov (1969) with \( A_0 + \gamma C = 1 \).

Returning to the determination of the creep strain rate tensor according to Eq. (12) and using Eqs. (13)–(16) and (24) together with the following relations:

\[
\frac{\partial \varphi_e}{\partial \chi_{kl}} = \frac{\partial \chi_0}{\partial \chi_{kl}} + \gamma C \frac{\partial \chi_1}{\partial \chi_{kl}}, \quad \frac{\partial \chi_0}{\partial \chi_{kl}} = \frac{A \chi_{kl} + \beta B I_1 \delta_{ki}}{\chi_0}, \quad \frac{\partial \chi_1}{\partial \chi_{kl}} = n_k n_l
\]  

(27)

one obtains the following expression:

\[
\dot{\chi}_{kl} = 2\lambda \varepsilon_e \left( \frac{A \chi_{kl} + \beta B I_1 \delta_{ki}}{\chi_0} + \gamma C n_k n_l \right).
\]  

(28)

By multiplying the right- and left-hand sides of Eq. (28) by \( \chi_{kl} \) and summing, one defines the mixed invariant given by expression (10) as follows:
Equating Eqs. (11) and (29), one finds that
\[ 2\lambda \varepsilon_c = \dot{\varepsilon}_c. \] (30)
Thus, substituting Eq. (30) into Eq. (28), one immediately arrives at the following constitutive equation:
\[ \dot{\varepsilon}_{kl} = \dot{\varepsilon}_c \left( \frac{A \varepsilon_{kl} + \beta B \varepsilon_{kl}}{\varepsilon_0} + \gamma \varepsilon_{nk} n_l \right) \] (31)
of the theory for creep deformation with anisotropic hardening for initially isotropic materials with different behaviors in tension and compression.

The equivalent creep strain rate \( \dot{\varepsilon}_c \) in Eq. (31) must be defined under the considered assumption of the type of the anisotropic hardening on the basis of data from basic experiments. Thus, considering kinematic hardening, one obtains specifically from Eq. (9) that
\[ \varepsilon_c = \varphi(\dot{\varepsilon}_c). \] (32)
One then assumes that the inverse relation also exists
\[ \dot{\varepsilon}_c = v(\varepsilon_c). \] (33)
The function \( v(\varepsilon_c) \) may be determined through the creep curves in one of the following forms: the Norton-type power relation
\[ v(\varepsilon_c) = \varepsilon_c^m \] (34)
the Prandtl-type hyperbolic sine law
\[ v(\varepsilon_c) = \sinh(\varepsilon_c/f) \] (35)
the Dorn-type exponential relation
\[ v(\varepsilon_c) = \exp(\varepsilon_c/g) \] (36)
and the Garofalo-type hyperbolic sine rule
\[ v(\varepsilon_c) = \left[ \sinh(\varepsilon_c/\mu) \right]^k, \] (37)
where \( m, f, g, \mu \) and \( k \) are some temperature-dependent material parameters. The creep approximations (34), (35) and (37) can be used for all values of the equivalent effective stress. In these cases, the following equation takes place
\[ v(0) = 0. \] (38)
On the other hand, the Dorn-type exponential relation (36) is not satisfied through condition (38), and therefore the possibility of its application for small values of the equivalent effective stress is questionable. The function
\[ v(\varepsilon_c) = (\varepsilon_c - \Sigma)^e \] (39)
can be used for all values of \( \varepsilon_c > \Sigma \) while the function
\[ v(x_e) = \left( \frac{x_e - \Sigma_1}{\Sigma_2 - x_e} \right)^r \]  

(40)

can be taken when the equivalent effective stress varies in the interval \( x_e \in (\Sigma_1, \Sigma_2) \). Here \( v, r, \Sigma, \Sigma_1, \Sigma_2 \) are some temperature-dependent material parameters. The use of Eqs. (39) and (40) is related to the condition that there is no creep process when \( x_e \leq \Sigma \), and when \( x_e \leq \Sigma_1 \), respectively. The case with \( x_e \geq \Sigma_2 \) corresponds to the instantaneous rupture and cannot be considered under application of the function given by Eq. (40).

Note that the functions given by Eqs. (34)–(37) and (39) under condition \( x_e \equiv \sigma_i \) are used in the classical theories (Boyle and Spence, 1983; Rabotnov, 1969), and the functions under condition \( x_e = z_i \) are used in the theory (Malinin and Khadjinsky, 1972) of creep deformation with kinematic hardening for the Huber–von Mises-type materials with the same properties in tension and compression. Functions given by Eqs. (34)–(37), (39) and (40) under condition (26) are used in the creep theory (Betten et al., 1998) with isotropic hardening.

2.3. Kinematic hardening rule

One now considers the formulation of the kinematic hardening rule for materials with different behaviors in tension and compression under multiaxial loading. In other words, one writes the evolution equation for the components of the back stress tensor that defines the translation of the creep potential surface without changing its initial shape and size. First, one returns to the definition of the creep part \( \psi_2(X_{kl}) \) in the Helmholtz free specific energy and one assumes

\[ \psi_2 = \frac{1}{2\rho} x_e^2. \]  

(41)

Here \( x_e \) is determined as the equivalent variable for \( X_{kl} \) \( (X_e \geq 0) \). Obviously, it should be a scalar function of the invariants of the tensor \( X_{kl} \) and should accept different values under different modes of loading (uniaxial tension, uniaxial compression and pure shear). In this regard, it is necessary to write the expression for the equivalent variable \( x_e \) based on three independent in a general case material parameters and three invariants for the tensor \( X_{kl} \) and the eigenvector \( n \). There are different ways to incorporate this equivalent variable. Without loss of generality, one introduces the following expression for \( x_e \):

\[ x_e = \sqrt{A_1 I_2 + B_1 I_1^2 + C_1 X_1} \]  

(42)

with invariants

\[ I_1 = X_{kl} \delta_{kl}, \quad I_2 = X_{kl} X_{kl}, \quad X_1 = X_{kl} n_k n_l. \]  

(43)

Here \( A_1, B_1 \) and \( C_1 \) are some temperature-dependent material parameters, \( \beta_1 \) and \( \gamma_1 \) are numerical coefficients which take into account the specific weight for the odd invariants in expression (42). It is seen that the equivalent variable \( x_e \) in Eq. (42) is written in a form similar to that in expression (24). The coefficients \( \beta_1 \) and \( \gamma_1 \) are
similar to the coefficients $\beta$ and $\gamma$ in expression (24), and they are introduced in expression (42) for convenience only. Of course, it is impossible to find the coefficients $\beta_1$ and $\gamma_1$ separately from parameters $A_1$ and $B_1$, respectively.

According to the definition of the back stress tensor

$$\rho_{kl} = \rho \frac{\partial \psi}{\partial X_{kl}}$$

(44)

and using Eq. (44) together with Eqs. (41)–(43), one obtains

$$\rho_{kl} = X_e \left( \frac{A_1 X_{kl} + \beta_1 B_1 \delta_{kl}}{X_0} + \gamma_1 C_1 n_i n_l \right),$$

(45)

where

$$X_0 = \sqrt{A_1 \bar{I}_2 + \beta_1 B_1 \bar{I}_1^2}.$$  

(46)

According to the idea originally suggested by Bailey (1926) and Orowan (1946), the creep deformation may be explained on the basis of the interaction between strain hardening and thermal softening (recovery). The strain hardening related to immobilization and piling up of dislocations at a barrier, causes an increase in the dislocation density. In this regard, it prevails in the first stage of the creep process and therefore hence the creep rate decreases. The thermal softening connected with escape of dislocations from their glide planes via slip and climb, involves the dislocation structure in time. Therefore in the second stage of the creep process the equilibrium between strain hardening and thermal softening is established. The minimum rate of creep depends upon the velocity with which the process of recovery takes place. Thus, one can write the following formula (Lemaitre and Chaboche, 1990; Malinin and Khadjinsky, 1972):

$$\dot{X}_{kl} = \dot{\varepsilon}_{kl} - G_{kl}$$

(47)

containing two terms: the kinematic hardening term which involves the creep strain rate tensor and a recall term $G_{kl}$ which provides a fading memory effect of the deformation path.

By analogy with the potential relation (12) together with Eqs. (24), (30) and (33) one can assume in accordance with the recommendation of Malinin and Khadjinsky (1972) the following representation:

$$G_{kl} = v(\rho_e) \frac{\partial \rho_e}{\partial \rho_{kl}}.$$  

(48)

Here the expression for the equivalent back stress in describing thermal softening is given by

$$\rho_e = \sqrt{A_2 J_2 + \beta_2 B_2 \bar{J}_1^2 + \gamma_2 C_2 \rho_1},$$

(49)

where $\rho_e \geq 0, J_1$ is the first invariant of the back stress tensor, $J_1 = \rho_{kl} \delta_{kl}, J_2$ is the second invariant of the back stress tensor, $J_2 = \rho_{kl} \rho_kl, \rho_1$ is the joint invariant of the
back stress tensor and eigenvector associated with the maximum principal Cauchy stress, $\rho_1 = \rho_{kl} n_k n_l$, $A_2$, $B_2$ and $C_2$ are some temperature-dependent material parameters, and $\beta_2$ and $\gamma_2$ are numerical coefficients which are introduced by analogy with the coefficients $\beta$, $\gamma$, and $\beta_1$, $\gamma_1$, and which take into account the specific weight for the odd invariants in expression (49). For example, putting in Eq. (49) $A_2 = \frac{1}{2}$, $B_2 = -\frac{1}{4}$, $\gamma_2 = 0$, one arrives at the expression for the equivalent back stress $\rho_0 = \rho_1$ in the theory (Malinin and Khadzinsky, 1972) for creep deformation with kinematic hardening for the Huber–von Mises-type materials with the same properties in tension and compression. Here $\rho_1$ is the back stress intensity, $\rho_1 = \sqrt{\frac{3}{2} \rho_{kl} \delta_{kl}}$, and $\rho_{kl}$ is the back stress deviator, $\rho_{kl} = \rho_{kl} - \frac{1}{2} J_1 \delta_{kl}$. Using Eqs. (48) and (49), and introducing the following notation:

$$
\rho_0 = \sqrt{A_2 J_2 + \beta_2 B_2 J_1^2}
$$

(50)

one obtains the following expression:

$$
G_{kl} = v(\rho_e) \left( \frac{A_2 \rho_{kl} + \beta_2 B_2 J_1 \delta_{kl}}{\rho_0} + \gamma_2 C_2 n_k n_l \right).
$$

(51)

Thus, the kinematic hardening rule for the materials under consideration is given by Eqs. (45), (47) and (51) where the function $(v \rho_e)$ is defined analogously with one of Eqs. (34)–(37), (39) and (40). The construction of the tensor function $\rho_{kl}$ in Eq. (45) and tensor function $G_{kl}(\rho_{mn}, n_m)$ in Eqs. (47) and (51) gives the opportunity to describe different strain hardening and different thermal softening under uniaxial tension, uniaxial compression and pure torsion.

2.4. Thermal dissipation

By considering the heat received by conduction through the boundary of the body, one can assume the existence of a thermal dissipation potential and then one can obtain the classical law of heat transfer or Fourier’s law (Lemaitre and Chaboche, 1990):

$$
\mathbf{q} = -k_0 \nabla T.
$$

(52)

Here $k_0$ is the coefficient of thermal conductivity, and $\mathbf{q} = (q_k)_{k=1}^3$ is the heat flux vector with its dual variable

$$
\mathbf{g} = \nabla T.
$$

(53)

2.5. The Clausius–Duhem inequality

The first aspect of the thermodynamic framework is concerned with three independent dissipation potentials considered in order to formulate the kinetic equations describing the evolution of creep deformation, creep hardening and heat. The second aspect of the thermodynamic framework is concerned with the second principle of thermodynamics to ensure the validity of the constitutive model proposed above. In this regard, one uses the Clausius–Duhem inequality
\[
\Phi = \sigma_{kl}\dot{\varepsilon}_{kl} - \rho_{kl}\dot{X}_{kl} - \frac{g_n\rho_n}{T} \geq 0. \tag{54}
\]

Using Eqs. (8), (47) and (51), and the equality \(\rho_{kl}G_{kl} = v(\rho_e)\rho_e\), one can express the first two terms in Eq. (54) as follows:

\[
\sigma_{kl}\dot{\varepsilon}_{kl} - \rho_{kl}\dot{X}_{kl} = z_{kl}\dot{\varepsilon}_{kl} + v(\rho_e)\rho_e. \tag{55}
\]

Considering Eqs. (10), (33) and (49), and the natural conditions \(L \geq 0, \nu_e \geq 0, \rho_e \geq 0\), one can obtain from Eq. (55) the inequality:

\[
\sigma_{kl}\dot{\varepsilon}_{kl} - \rho_{kl}\dot{X}_{kl} \geq 0. \tag{56}
\]

Using then Eqs. (52)–(54), one can finally express the third term in Eq. (54) as follows:

\[
-\mathbf{g} \cdot \mathbf{q} = \frac{k_0}{T} |\text{grad} T|^2 \geq 0. \tag{57}
\]

Thus, within the proposed model the second thermodynamic principle is always valid.

3. Basic experiments

One now considers a method for determining the 10 material parameters in Eqs. (31), (33), (45), (47) and (51) on the basis of Eq. (34) as well as of the same representation for function \(v(\rho_e)\). These parameters are: exponent \(m\) in Eq. (34), parameters \(A, B, C\) in Eq. (24), \(A_1, B_1, C_1\) in Eq. (42), and parameters \(A_2, B_2, C_2\) in Eq. (49). In order to do this, one needs the results from basic experiments in uniaxial tension, uniaxial compression and pure torsion taken from standard specimens in which constant stresses are realized.

Let the following relations hold for the case of uniaxial tension (\(\sigma_{11} > 0\)):

\[
\dot{\varepsilon}_{11} = A_+(\sigma_{11} - \rho_{11})^m, \quad \dot{\rho}_{11} = B_+\dot{\varepsilon}_{11} - C_+\rho_{11}^m. \tag{58}
\]

Considering uniaxial compression (\(\sigma_{11} < 0\)), one has

\[
\dot{\varepsilon}_{11} = -A_-|\sigma_{11} - \rho_{11}|^m, \quad \dot{\rho}_{11} = -B_-|\dot{\varepsilon}_{11}| + C_-|\rho_{11}|^m. \tag{59}
\]

One assumes that for the case of pure torsion (\(\sigma_{12} \neq 0\)) one has the following relations:

\[
2\dot{\varepsilon}_{12} = A_0(\sigma_{12} - \rho_{12})^m, \quad \dot{\rho}_{12} = 2B_0\dot{\varepsilon}_{12} - C_0\rho_{12}^m. \tag{60}
\]

Here \(m, A_+, B_+, C_+, A_-, B_-, C_-, A_0, B_0, C_0\) are temperature-dependent material constants, and \(A_+ > 0, B_- > 0, C_- > 0\). Eqs. (58)–(60) are written for basic experiments within the Bailey–Orowan creep theory (Bailey, 1926; Orowan, 1946). Material constants in Eqs. (58)–(60) can be found on the basis of fitting the experimental data using modern methods of parameter identification (Mahnken and Stein, 1996). In a general case one assumes that a material constant \(m\) does not depend on the kind of loading, and \(A_+ \neq A_-\), \(B_+ \neq B_-\), \(C_+ \neq C_-\).
Next, let one consider Eqs. (31), (33), (45), (47) and (51) on the basis of Eq. (34) as well as of the same representation for function \( v(\rho_c) \) in the case of these basic experiments. In the case of uniaxial tension Eqs. (31), (33), (45), (47) and (51) reduce to

\[
\begin{align*}
\dot{\varepsilon}_{11} &= (\sqrt{A + \beta B + \gamma C})^{m+1}(\sigma_{11} - \rho_{11})^m, \\
\dot{\rho}_{11} &= (\sqrt{A_1 + \beta_1 B_1 + \gamma_1 C_1})^2\dot{\varepsilon}_{11} - (\sqrt{A_1 + \beta_1 B_1 + \gamma_1 C_1})^2(\sqrt{A_2 + \beta_2 B_2 + \gamma_2 C_2})^{m+1}\rho_{11}^m.
\end{align*}
\]  

(61)

Comparing Eqs. (58) and (61), one obtains

\[
\begin{align*}
\sqrt{A + \beta B + \gamma C} &= A_+^{1/(m+1)}, \\
\sqrt{A_1 + \beta_1 B_1 + \gamma_1 C_1} &= \sqrt{B_+}, \\
\sqrt{A_2 + \beta_2 B_2 + \gamma_2 C_2} &= (C_+/B_+)^{1/(m+1)}.
\end{align*}
\]  

(62)

Similarly, considering uniaxial compression and pure torsion, and comparing relations resulting from Eqs. (31), (33), (45), (47) and (51) with relations from Eqs. (59) and (60), respectively, one obtains

\[
\begin{align*}
\sqrt{A + \beta B} &= A_-^{1/(m+1)}, \\
\sqrt{A_1 + \beta_1 B_1} &= B_-, \\
\sqrt{A_2 + \beta_2 B_2} &= (C_-/B_-)^{1/(m+1)}
\end{align*}
\]  

(63)

in the case of uniaxial compression and

\[
\begin{align*}
\sqrt{2A + \gamma C} &= A_0^{1/(m+1)}, \\
\sqrt{2A_1 + \gamma_1 C_1} &= \sqrt{2B_0}, \\
\sqrt{2A_2 + \gamma_2 C_2} &= (2C_0/B_0)^{1/(m+1)}
\end{align*}
\]  

(64)

in the case of pure torsion.

It is now easy to find from Eqs. (62)–(64) the material parameters in the proposed theory for creep deformation with kinematic hardening

\[
\begin{align*}
\gamma C &= A_+^{1/(m+1)} - A_-^{1/(m+1)}, \\
\sqrt{2A} &= A_0^{1/(m+1)} - \gamma C, \\
\beta B &= A_2^{2/(m+1)} - A, \\
\gamma_1 C_1 &= \sqrt{B_+} - \sqrt{B_-}, \\
\sqrt{2A_1} &= \sqrt{2B_0} - \gamma_1 C_1, \\
\beta_1 B_1 &= B_- - A_1, \\
\gamma_2 C_2 &= (C_+/B_+)^{1/(m+1)} - (C_-/B_-)^{1/(m+1)}, \\
\sqrt{2A_2} &= (2C_0/B_0)^{1/(m+1)} - \gamma_2 C_2, \\
\beta_2 B_2 &= (C_-/B_-)^{2/(m+1)} - A_2.
\end{align*}
\]  

(65)

The procedure considered here for the determination of the material parameters in Eqs. (31), (33), (45), (47) and (51) can be extended also to other representations \( v(\alpha_c) \) and \( v(\rho_c) \).

4. Comparison of theoretical and experimental results

The results generated from the proposed theory of creep deformation with kinematic hardening are compared in this section with those obtained from experiments under multiaxial loading with constant stresses, uniaxial non-proportional and multiaxial non-proportional loadings for both isothermal and nonisothermal
processes. The model parameters are first determined using the data from a series of basic experiments with constant stresses outlined in this paper.

4.1. Creep deformation under multiaxial loading with constant stresses for isothermal processes

The capability of the proposed theory with kinematic hardening is first demonstrated in order to describe the creep deformation for the case of constant stresses in

![Graphs showing creep curves for different stress levels under compression, tension, and torsion.](image)

Fig. 1. Creep curves of aluminum alloy AK4-1T at the temperature of 473 K under compression (a), tension (b) and torsion (c).
isothermal processes. One considers the creep behavior of the aluminum alloy AK4-1T at the temperature of 473 K (Gorev et al., 1978; Rubanov, 1987). The chemical composition of this light alloy is 2.2 Cu, 1.6 Mg, 0.55 Ni and 0.015 Ti in weight percentages. Creep tests were conducted on specimens taken from the three symmetry directions of a 40-mm thick plate as well as on specimens oriented at an angle of \(\pi/4\) with respect to the longitudinal and transverse directions. Basic experiments in the normal direction of the plate were conducted on cylindrical specimens of 12 mm diameter and a 20 mm gage length. All other specimens for testing were thin-walled tubular specimens of 20-mm outside diameter, 1-mm wall thickness and 35-mm gage length. It was established (Gorev et al., 1978; Rubanov, 1987) that the given light alloy may be considered as an initially isotropic compressible material under creep conditions. Figs. 1(a)–(c) show creep curves of an aluminum alloy under

![Creep curves](image)

**Fig. 2. Growth of the specific dissipation energy for aluminum alloy AK4-1T at the temperature of 473 K under combined tension with torsion.**
uniaxial compression (a), uniaxial tension (b) and pure torsion (c) for different values of the stresses pointed out here. Experimental data are denoted by circles while solid lines represent theoretical results based on Eqs. (58)–(60) and using the values for the material constants given in Eq. (A.1). It is not difficult to see from Figs. 1(a) and (b) that the creep strain rate when the specimen is subjected to tension is more than twice the analogous magnitude when the specimen is subjected to compression. Furthermore, at the same absolute value of the Cauchy stress intensity given by expression (18) and at any given time, the creep strain intensity is largest under pure torsion and smallest under uniaxial compression.

Fig. 3. Growth of the specific dissipation energy for aluminum alloy AK4-1T at the temperature of 473 K under combined compression with torsion.
Fig. 4. Growth of the specific dissipation energy for aluminum alloy AK4-1T at the temperature of 448 K (b) under uniaxial compression for the loading program (a).

Fig. 5. Growth of the specific dissipation energy for aluminum alloy AK4-1T (c) under uniaxial tension for the loading program (b) in nonisothermal process (a) with numerous variations in temperature from 423 to 448 K.
One now considers the experimental data for the aluminum alloy AK4-1T under multiaxial loading. Creep tests were carried out on thin-walled tubular specimens loaded by a torsional moment and an axial (compressive or tensile) force. Figs. 2(a)–(c) and 3(a)–(c) show the change in the specific dissipation energy

\[
\omega = \int \sigma_{kl} \dot{e}_{kl} \, dt
\]  

(66)

with respect to time for various values of stresses \(\sigma_{11}\) and \(\sigma_{12}\). Here symbols represent experimental data while solid lines represent results of calculations using Eqs. (31), (33), (34), (45), (47), (51) and (65) with material constants given in Eq. (A.1).

4.2. Creep deformation under non-proportional uniaxial and non-proportional multiaxial loadings for both isothermal and nonisothermal processes

The capability of the proposed theory with kinematic creep hardening is secondly demonstrated to describe the creep deformation under non-proportional uniaxial and non-proportional multiaxial loadings for both isothermal and nonisothermal processes.

Fig. 6. Growth of the specific dissipation energy for aluminum alloy AK4-1T (c) under pure torsion for the loading program (b) in nonisothermal process (a) with linear variation in temperature.
For this purpose one will consider again the creep deformation of the aluminum alloy AK4-1T (Gorev et al., 1978; Rubanov, 1987). The same material was used earlier to study the creep behavior for the case of constant stresses at the temperature of 473 K. Creep curves of the given material at other temperature levels (Rubanov, 1987) under uniaxial tension, uniaxial compression, and pure torsion for different values of constant stress can be also described on the basis of Eqs. (58)–(60) with the values for the material constants given in Eqs. (A.2) and (A.3). Note that for the aluminum alloy AK4-1T at the temperature of 448 K the difference in the creep strain rate between tension and compression in the secondary stage is five times while at the temperature of 423 K the difference is 12.5 times.

One now considers the experimental data for the aluminum alloy AK4-1T under non-proportional loading conditions (Rubanov, 1987). Thin-walled tubes with an outside diameter of 20 mm, a wall thickness of 1 mm, and a gage length of 35 mm are

![Fig. 7. Creep curves for aluminum alloy AK4-1T (d) obtained for nonproportional loading of the specimen that was subjected to torque (c) and to axial loading by tensile force (b) in nonisothermal conditions (a).](image-url)
subjected to non-proportional loading by an axial (tensile or compressive) force together with (or without) torque.

First, one considers the creep deformation of the aluminum alloy under uniaxial non-proportional loading for isothermal process (Fig. 4) and for nonisothermal process (Fig. 5) as well as under non-proportional loading by torque for the non-isothermal process (Fig. 6). Circles in Figures represent results of the tests for the aluminum alloy under consideration in the form of dependence \( \frac{x}{C_0 t} \) for various values of the stress \( \sigma_{11}(t) \) (or \( \sigma_{11}(t) \)) and temperature \( T(t) \), while solid lines here correspond to results of calculations on the basis of Eqs. (31), (33), (34), (45), (47), (51), (65), (66), (A.4) and (A.5).

Fig. 8. Creep curves for aluminum alloy AK4-1T (d) obtained for nonproportional loading of the specimen that was subjected to torque (c) and to axial loading by compressive force (b) in nonisothermal conditions (a).
Next, one considers the creep deformation of the given material under multiaxial nonproportional loading for nonisothermal processes (Figs. 7 and 8). Thin-walled tubular specimens were subjected to torque, while an axial loading (tensile force in Fig. 7(b) and compressive force in Fig. 8(b)) was applied simultaneously. Experimental data are represented here in the form of dependencies $\varepsilon_{11} - t$ (circles) and $2\varepsilon_{12} - t$ (squares) for various values of stresses $\sigma_{11}(t)$ and $\sigma_{12}(t)$, and temperature $T(t)$. The dashed line and the solid line indicate theoretical results for shear strains and axial strains, respectively.

By considering the unique scatter of experimental data for creep deformation, the agreement between the theoretical results and test results under uniaxial non-proportional and multiaxial non-proportional loadings, and for both isothermal and nonisothermal conditions may be considered satisfactory.

5. Conclusions

A new theory of creep deformation with kinematic hardening is proposed for initially isotropic materials with different properties in tension and compression within the thermodynamic formulation. This theory is based on the definition of three potentials (creep, hardening and recovery), each of them written using three irreducible invariants, as well as, three independent material characteristics obtained from uniaxial tension tests, uniaxial compression tests, and pure torsion tests for different values of constant stress at constant temperatures. A satisfactory agreement is obtained between the theoretical results and the experimental data for the creep deformation under multiaxial loading with constant stresses for isothermal processes as well as under uniaxial non-proportional and multiaxial non-proportional loadings for both isothermal and nonisothermal conditions. The description of additional physical phenomena occurring in initially isotropic materials, such as viscoplasticity, combined isotropic and anisotropic creep hardening, cyclic creep, ratcheting etc. will be discussed in a forthcoming paper.

Acknowledgements

The research described in this paper is sponsored by the Alexander von Humboldt Foundation, Germany, the National Research Council of the USA and the Japan Society for the Promotion of Science (Long-Term Program).

Appendix A. Material constants

The material constants in Eqs. (58)–(60) for the aluminum alloy AK4-1T have the following values:
\(m = 8, \quad A_+ = 1.00 \times 10^{-22} \text{ MPa}^{-m} \text{ h}^{-1}, \quad B_+ = 7.00 \times 10^4 \text{ MPa,}
\)
\(C_+ = 3.50 \times 10^{-10} \text{ MPa}^{1-m} \text{ h}^{-1},
\)
\(A_- = 1.43 \times 10^{-22} \text{ MPa}^{-m} \text{ h}^{-1}, \quad B_- = 2.50 \times 10^4 \text{ MPa,}
\)
\(C_- = 7.50 \times 10^{-14} \text{ MPa}^{1-m} \text{ h}^{-1},
\)
\(A_0 = 7.25 \times 10^{-20} \text{ MPa}^{-m} \text{ h}^{-1}, \quad B_0 = 1.75 \times 10^4 \text{ MPa,}
\)
\(C_0 = 2.59 \times 10^{-11} \text{ MPa}^{1-m} \text{ h}^{-1}
\)

at the temperature of \(T = 473\) K,

\(m = 13, \quad A_+ = 1.80 \times 10^{-35} \text{ MPa}^{-m} \text{ h}^{-1}, \quad B_+ = 2.00 \times 10^4 \text{ MPa,}
\)
\(C_+ = 3.50 \times 10^{-3} \text{ MPa}^{1-m} \text{ h}^{-1},
\)
\(A_- = 5.00 \times 10^{-36} \text{ MPa}^{-m} \text{ h}^{-1}, \quad B_- = 8.00 \times 10^3 \text{ MPa,}
\)
\(C_- = 1.00 \times 10^{-13} \text{ MPa}^{1-m} \text{ h}^{-1},
\)
\(A_0 = 1.00 \times 10^{-31} \text{ MPa}^{-m} \text{ h}^{-1}, \quad B_0 = 1.20 \times 10^4 \text{ MPa,}
\)
\(C_0 = 4.00 \times 10^{-6} \text{ MPa}^{1-m} \text{ h}^{-1}
\)

at the temperature of \(T = 448\) K, and

\(m = 25, \quad A_+ = 5.00 \times 10^{-65} \text{ MPa}^{-m} \text{ h}^{-1}, \quad B_+ = 5.50 \times 10^4 \text{ MPa,}
\)
\(C_+ = 2.00 \times 10^{-29} \text{ MPa}^{1-m} \text{ h}^{-1},
\)
\(A_- = 1.00 \times 10^{-36} \text{ MPa}^{-m} \text{ h}^{-1}, \quad B_- = 8.00 \times 10^3 \text{ MPa,}
\)
\(C_- = 1.00 \times 10^{-13} \text{ MPa}^{1-m} \text{ h}^{-1},
\)
\(A_0 = 1.00 \times 10^{-31} \text{ MPa}^{-m} \text{ h}^{-1}, \quad B_0 = 1.20 \times 10^4 \text{ MPa,}
\)
\(C_0 = 4.00 \times 10^{-6} \text{ MPa}^{1-m} \text{ h}^{-1}
\)

at the temperature of \(T = 423\) K. It is possible to obtain from the experimental data in Eqs. (A.1)–(A.3) the following approximations:

\(m = m_1 T^2 + m_2 T + m_3, \quad A_+/Q_1 = \exp \left( a_1^+ T^2 + a_2^+ T + a_3^+ \right),
\)
\(A_-/Q_1 = \exp \left( a_1^- T^2 + a_2^- T + a_3^- \right), \quad A_0/Q_1 = \exp \left( a_0^+ T^2 + a_0^- T + a_0^0 \right),
\)
\(B_+/Q_2 = \exp \left( b_1^+ T^2 + b_2^+ T + b_3^+ \right), \quad B_-/Q_2 = \exp \left( b_1^- T^2 + b_2^- T + b_3^- \right),
\)
\(B_0/Q_2 = \exp \left( b_0^+ T^2 + b_0^- T + b_0^0 \right), \quad C_+/Q_3 = \exp \left( c_1^+ T^2 + c_2^+ T + c_3^+ \right),
\)
\(C_-/Q_3 = \exp \left( c_1^- T^2 + c_2^- T + c_3^- \right), \quad C_0/Q_3 = \exp \left( c_0^+ T^2 + c_2^+ T + c_3^0 \right),
\)
\(Q_1 = 1 \text{ MPa}^{-m} \text{ h}^{-1}, \quad Q_2 = 1 \text{ MPa,} \quad Q_3 = 1 \text{ MPa}^{1-m} \text{ h}^{-1},
\)
\(m_1 = 0.0056 \text{ K}^{-2}, \quad m_2 = -5.3576 \text{ K}^{-1}, \quad m_3 = 1289.26,
\)
\(a_1^+ = -0.030969 \text{ K}^{-2}, \quad a_2^+ = 29.6954 \text{ K}^{-1}, \quad a_3^+ = -7168.14,
\)
\(b_1^+ = 0.00181 \text{ K}^{-2}, \quad b_2^+ = -1.61827 \text{ K}^{-1}, \quad b_3^+ = 371.316,
\[ c_1^+ = -0.06124 \, \text{K}^{-2}, \quad c_2^+ = 55.7536 \, \text{K}^{-1}, \quad c_3^+ = -12692.0, \]
\[ a_1^- = -0.031667 \, \text{K}^{-2}, \quad a_2^- = 30.4042 \, \text{K}^{-1}, \quad a_3^- = -7346.76, \]
\[ b_1^- = 0.0014144 \, \text{K}^{-2}, \quad b_2^- = -1.25712 \, \text{K}^{-1}, \quad b_3^- = 288.293, \]
\[ c_1^- = -0.0026267 \, \text{K}^{-2}, \quad c_2^- = 2.40771 \, \text{K}^{-1}, \quad c_3^- = -581.393, \]
\[ a_1^0 = -0.0027889 \, \text{K}^{-2}, \quad a_2^0 = 26.7778 \, \text{K}^{-1}, \quad a_3^0 = -6470.47, \]
\[ b_1^0 = -0.00131 \, \text{K}^{-2}, \quad b_2^0 = 1.22168 \, \text{K}^{-1}, \quad b_3^0 = -274.981, \]
\[ c_1^0 = -0.02669 \, \text{K}^{-2}, \quad c_2^0 = 24.1036 \, \text{K}^{-1}, \quad c_3^+ = -5454.05. \]


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