A Study of the Static Structural Limitations of Nonwelded Chainmail

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Problem Description

• Chainmail is a flexible assembly of rigid metal rings
• A vertically suspended sheet of chainmail is subject to static load based on its own weight
• Structural integrity depends on the rings in the uppermost row
Approach

• Model rings as curved beams of circular cross-section subjected to radial point loading
  – Consider both elastic and plastic response
• Use classical curved beam deflection theory to determine worst-case cut orientation
• Verify results with 3D ANSYS finite element models
Classical Modeling

- Ring loading is consistent with that of an arch subject to a radial load, with some special considerations
  - Force W is always applied 180° from Point B
  - Point A has an associated mobile coordinate system that varies with arch section size $2\theta$
Adaptation of Classical Modeling

180°
Adaptation of Classical Modeling

270°
Effect of Cut Orientation

\[ \delta_{\text{critical}} = d - K = \begin{cases} \sec \left( \frac{2\theta - 180^\circ}{2} \right) \frac{\delta_j}{W} & \text{for } 180^\circ \leq 2\theta < 270^\circ, \\ \csc \left( \frac{2\theta - 180^\circ}{2} \right) \frac{\delta_j}{W} & \text{for } 270^\circ \leq 2\theta \leq 360^\circ. \end{cases} \]
Governing Equations

\[ \frac{\delta_{A_j}}{W} = \frac{-R^3}{EI} LF_j \]

\[ \frac{\delta_{A_i}}{W} = \frac{-R^3}{EI} LF_i \]

\[ \frac{\Psi_A}{W} = \frac{-R^2}{EI} LF_M \]

\[ \frac{LF_j}{W} = \frac{k_1}{2} (\theta \sin(\phi) + \phi \sin(\phi) - \sin(\theta) \cos(\theta) \sin(\phi) - \sin^2(\theta) \cos(\phi)) + k_2 (\cos(\phi) - \cos(\theta)) \]

\[ \frac{LF_i}{W} = \frac{k_1}{2} (\theta \cos(\phi) + \phi \cos(\phi) + \sin(\theta) \cos(\theta) \cos(\phi) + \cos^2(\theta) \sin(\phi)) + k_2 (\sin(\theta) + \sin(\phi) - 2 \sin(\theta) \cos(\theta) \cos(\phi) - 2 \cos^2(\theta) \sin(\phi)) \]

\[ \frac{LF_M}{W} = k_2 (1 + \sin(\theta) \sin(\phi) - \cos(\theta) \cos(\phi)) \]
Effect of Cut Orientation
Elastic-Plastic Model

- An 18ga ¼” ID Al-5356 ring yields at 2.116 lbf in worst-case orientation
- Considering the strain-hardening behavior of the material, the ring will reach unweaving deflection when 2.161 lbf are applied
  - 13,370 rings
    - Roughly 4.7 ft²
Critical Load for Various Ring Geometries (Al-5356)
FEA Results

- Model setup: applied calculated force, examined deflection results
- FEA results agree well with hand calculations
Questions?