A Comparison of the Predicted Mechanical Behavior of Lug Joints using Strength of Materials Models and Finite Element Analysis

by
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LIST OF SYMBOLS

Fbru = Lug Ultimate Bearing Stress (psi)
Fbry = Lug Yield Bearing Stress (psi)
Ftu = Ultimate Tensile Strength (psi)
Fty = Yield Tensile Strength (psi)
Fbru = Allowable Ultimate Bearing Stress (psi)
Fbry = Allowable Yield Bearing Stress (psi)
Ftu = Ultimate Tensile Stress (psi)
Fnu = Allowable Lug Net-Section Tensile Ultimate Stress (psi)
Fay = Allowable Lug Net-Section Tensile Yield Stress (psi)
Fbry = Allowable Bearing Yield Stress for Bushings (psi)
Fcy,B = Bushing Compressive Yield Stress (psi)
Fbru,B = Allowable Bearing Ultimate Stress for Bushings (psi)
Fsu,p = Ultimate Shear Stress of Pin Material
Ftu,p = Pin Ultimate Tensile Strength (psi)
Pnu = Allowable Lug Net-Section Ultimate Load (lb)
Pu,B = Allowable Bushing Ultimate Load (lb)
Pu,LB = Allowable Lug/Bushing Ultimate Load (lb)
Pus,p = Pin Ultimate Shear Load (lb)
Pub,p = Pin Ultimate Bending Load (lb)
Pub,p,max = Balanced Design Pin Ultimate Bending Load (lb)
P_T = Lug Tang Strength (lb)
Pall = Allowable Joint Ultimate Load (lb)
Ptru = Allowable Lug Transverse Ultimate Load (lb)
Pru,B = Allowable Bushing Transverse Ultimate Load (lb)
Mmax,p = Maximum Pin Bending Moment (in-lb)
Mu,p = Ultimate Pin Failing Moment (in-lb)
K = Allowable Load Coefficient
Kn = Net-Section Stress Coefficient
kbp = Plastic Bending Coefficient for the Pin
kbT = Plastic Bending Coefficient for the Tang
$K_{\text{tru}} = \text{Transverse Ultimate Load Coefficient}$

$K_{\text{try}} = \text{Transverse Yield Load Coefficient}$

$a = \text{Distance from the Edge of the Hole to the edge of the Lug (in)}$

$b = \text{Effective bearing Width (in)}$

$D = \text{Hole Diameter (in)}$

$D_p = \text{Pin Diameter (in)}$

$E = \text{Modulus of Elasticity (psi)}$

$e = \text{Edge Distance (in)}$

$P = \text{Load (lb)}$

$g = \text{Gap between Lug and Link (in)}$

$h_1..h_4 = \text{Edge Distances in Transversely Loaded Lug (in)}$

$h_{av} = \text{Effective Edge Distance in Transversely Loaded Lug (in)}$

$t_{\text{lug}} = \text{Lug Thickness (in)}$

$t_{\text{link}} = \text{Link Thickness (in)}$

$w_{\text{lug}} = \text{Lug Width (in)}$

$w_{\text{link}} = \text{Link Width (in)}$

$\varepsilon = \text{Strain (in/in)}$

$\varepsilon_{\text{engineering}} = \text{Engineering Strain (in/in)}$

$\sigma_{\text{engineering}} = \text{Engineering Stress (psi)}$

$\sigma_{\text{true}} = \text{True Stress (psi)}$

**Subscripts:**

$B = \text{bushing}$

$p = \text{pin}$

$\text{lug} = \text{lug}$

$\text{link} = \text{link}$

$T = \text{tang}$
ACKNOWLEDGMENT

I would like to thank Alex Simpson, my coworker at Pratt & Whitney, for his continual guidance and teaching during the completion of this project and for providing me with the base macros from which my ANSYS models were created. I would also like to thank my friends and family for their support through the course of my education.
ABSTRACT

Historically, lug and pin joints have been designed based on theoretical strength of materials models and empirical data developed in the 1950’s and 1960’s. With the increasing usage of finite element analysis (FEA) codes, it is necessary to determine whether the results obtained from FE analysis concur with those historically acceptable values. For this study, geometry was determined and analyzed using the theoretical calculations to determine the ultimate joint load. Margin of safety for each joint component was calculated based on ultimate joint load. This geometry was then modeled in ANSYS using parametric macros. Mesh density and plasticity studies were undertaken to ensure that the FE model predicted stress and strain correctly and treated the joint components the same way that the theoretical calculation did. It was determined that the pin and bushings should be modeled with elastic material properties while the lug and link were modeled with true stress-strain curves and allowed to yield. Following these studies, a purely axially loaded and purely transversely loaded model were created and loaded to the ultimate load obtained from the strength of materials calculations. Margin of safety for each component was calculated using two methods, peak stress and stress averaged over the contact area. Using peak stress was very conservative and predicted margins were much less than those calculated from the theoretical calculations. Using average stress resulted in good correlation between the theoretical margin based on load and the FE margin based on stress. In most cases, the FE average stress margin of safety was less conservative than the margin predicted by the theoretical calculation; however, this difference was generally less than 10%. Thus, analyzing lug joints using FE codes leads to similar results as those predicted by strength of materials calculations.
1. Introduction

Lugs are connector type elements used as structural supports for pin connections. Prior to the 1950’s, lugs were overdesigned as weight and space were not design driving factors. With the tightening of weight, cost, and space requirements in the aerospace industry, a more precise method of lug analysis was required.

Early aerospace lug analysis, developed in the 1950’s at Lockheed Aircraft Corporation by F.P. Cozzone, M.A.. Melcon, and F.M Hoblit and summarized in Reference 2 and 3, addressed prior anticonservative assumptions, such as incomplete evaluation of the effect of stress concentration and pin adequacy with respect to bending, and focused on steel and aluminum alloys. This work provided analysts with a defined and experimentally validated approach to analyze axial, transverse, and oblique loads, the later which can be resolved into axial and transverse components. During the 1960’s, the United States Air Force issued a manual, Reference 1’s Stress Analysis Manual, that built upon Cozzone, Melcon, and Hoblit’s work and, to this day, remains the aerospace industry standard method for designing lug, link, and pin joints. Industry has since verified this method for other materials, such as nickel based alloys, titanium, and other heat resistant alloys. [1]

Both the Lockheed engineers and the Air Force stated that, in considering any lug-pin combination, all ultimate failure methods must be considered. These include (1) tension across the net section resulting from hole Kt, (2) shear tear-out or bearing, closely related failures based on empirical data, (3) hoop tension at the tip of the lug, which requires no additional analysis as shear tear-out and bearing allowables account for hoop tension failure, (4) shearing of the pin, (5) bending of the pin, which can lead to excessive pin deflection and the build up of load near the lug shear plane, and (6) excessive yielding of the bushing if one is used. For each possible failure method, the applied load is compared to the failure load (yield or ultimate) by means of a margin of safety calculation. [2]
With the ever increasing prevalence and usage of finite element analysis codes such as ANSYS, NASTRAN, and ABAQUS, it is necessary to determine whether the results obtained from FEA concur with those historically acceptable values generated from theoretical hand calculations.

Prior work in this field was completed as part of the National Transportation Safety Board (NTSB) investigation into the failure of the composite vertical tail of the American Airlines Flight 587 – Airbus A300-600R. This failure caused the plane to crash shortly after the November 12, 2001 takeoff, killing all 260 people onboard as well as five on the ground. Two structural teams were assembled at the NASA Langley Research Center to assist the NTSB. One team focused on the global deformation, load transfer, and failure load of the vertical tail and rudder. This team determined that, of the six laminate composite lugs that attached the tail to the aluminum fuselage through a pin and clevis connection, the right rear lug carried the largest load during the failure event compared to the design allowables. This team provided the associated loads and deformations to the second team, which focused solely on the lug failure. Figure 1 shows the lug location and geometry.

![Figure 1: Vertical Tail to Fuselage Attachment Points and Associated Lug Geometry [6]](image-url)
The detailed lug analysis team used two modeling approaches: (1) a solid-shell model and (2) a layered shell model. Both models were created in ABAQUS. The solid-shell model was built using 3D quadrilateral elements in the vicinity of the pin hole and shell elements in all other locations. Multi-point constraints were used to connect the solid and shell elements. In the layered shell model, the previously modeled 3D region was converted to 14 layers of quadrilateral shell elements that were connected by 3D decohesion elements. In both models, the pin was modeled as a rigid cylinder with its diameter equal to the diameter of the lug hole, meaning that no bushing existed in the assembly. As lug to pin friction data was not available, frictionless multi-point constraint equations were prescribed between the lug hole and the rigid surface to prevent rigid motion sliding of the pin. The displacement of the pin along its axis was set to the average of the lug hole displacements in that direction over a 120° arc that represented the contact region between the pin and lug. Material properties were set to degrade as the material failed under load. Displacements were obtained from the global model and applied to the lug model. [5]

To assess the validity of these models, they were first compared to each other and then to experimental testing. It was shown that both models predicted the same magnitude and spatial distribution of displacements as the original Airbus model under set loads. Thus, the solid-shell model was then compared to a certification test completed by Airbus in 1985 and subcomponent tested completed in 2003 as part of the failure investigation. The stiffness of the lug, the failure load, and the failure mechanism, and the failure location predicted by the model all agreed with the experimentally determined values from the 1985 test. Agreement was also obtained between the model and the 2003 subcomponent test and accident conditions. [5]

This paper will expand upon prior work to consider an isotropic material lug system under uniform axial and transverse load modeled using solely solid and contact elements. The ultimate goal of this work will be to show agreement between the theoretical analysis technique presented in the Stress Analysis Manual and the ANSYS FE analysis. This will be achieved by first completing the theoretical analysis for set
geometry and material properties. Next, a FE model of the assembly will be created by augmenting the automated lug FEA generator developed by Alex Simpson at Pratt & Whitney. This augmentation will include the addition of lug and link bushings and the addition of geometric model division so to simplify the extraction of model loads. Once the model is functional a mesh density study will be completed to ensure the convergence of stresses in the contact regions of the pin, bushings, and lugs. The results of this fully converged model will be compared with those obtained from the theoretical analysis. For a set load, the stress margins of the FEA will be compared to the load margins of the theoretical analysis.

As was stated in the introduction, one must consider numerous failure modes when assessing a lug joint. The following outlines the procedure for and theory behind lug analysis as presented in the Stress Analysis Manual, Reference 1. This analysis is based on basic strength of materials concepts as well as empirical data. The strength of materials equations on which the analysis is based calculate the average stress in plates and beams. Thus, peak stresses in the lug, link, bushings, and pin could be higher than those assumed in the following calculations. Empirical data has been collected from lab tests to determine load coefficients and modified strength allowables.

2.1 Lug, Bushing, and Pin Strength under Uniform Axial Loading

Lugs must be analyzed for bearing and net-section strength while pins are analyzed for shear and bending load. See Figure 2 below for an overview of basic lug geometry.

\[ \text{Figure 2: Lug Geometry for Uniform Axial Loading [1]} \]

2.1.1 Lug Bearing Stress under Uniform Axial Load

To calculate bearing stresses in the region of the lug and/or link forward of the net-section, one must first determine the allowable load coefficient (K) which is related to the ratio of e/D. When e/D is less than 1.5, lug failures are likely to result from shear tear-out or hoop tension while when e/D is greater than 1.5, failure due to bearing is more likely. In the majority of cases, failure depends on the interaction of several failure methods; however, K takes into account these interaction effects. Figure 3 depicts the relationship between e/D and K.
With the determination of $K$, one can calculate the lug ultimate bearing stress ($F_{bru}$) from the equations

$$F_{bru} = K \frac{a}{D} F_{tu} \quad \text{when } e/D < 1.5 \quad [1]$$

$$F_{bru} = K \cdot F_{ty} \quad \text{when } e/D \geq 1.5 \quad [2]$$

Additionally, the lug yield bearing stress ($F_{bry}$) is calculated from the equations

$$F_{bry} = K \frac{a}{D} F_{ty} \quad \text{when } e/D < 1.5 \quad [3]$$

$$F_{bry} = K \cdot F_{ty} \quad \text{when } e/D \geq 1.5 \quad [4]$$

Based on the strength of materials plate bearing stress calculations in which load capability is directly proportional to the area of the rectangle representing the projection of the pin on the plate section of the lug, the allowable lug ultimate bearing load ($P_{bru}$) for lug failure in bearing, shear tear-out, or hoop tension is

$$P_{bru} = F_{bru} \cdot D \cdot t \quad \text{when } F_{tu} \leq 1.304*F_{ty} \quad [5]$$

$$P_{bru} = 1.304 \cdot F_{bry} \cdot D \cdot t \quad \text{when } F_{tu} > 1.304*F_{ty} \quad [6]$$
It should be noted that the above equations apply only if the load is uniformly distributed across the lug. If the pin is flexible and bends, load will pile up near the lug shear planes, leading to possible premature failure. The procedure for taking this phenomenon into account will be outlined further in this chapter. [1]

2.1.2 Lug Net-Section under Uniform Axial Load

The allowable net-section tensile ultimate stress \( (F_{nu}) \) across the cross section of the hole is affected by the lug material’s ability to yield, thereby relieving the stress concentration at the edge of the hole. Similar to the bearing procedure above, one must first determine the net tension stress coefficient \( (K_n) \), see Figure 4, which is a function of lug geometry, ultimate and yield strength of the lug material, and strains of the lug material and is based on empirical data. [1]
Figure 4: Net Tension Stress Coefficient [1]

This constant scales the allowable net-section stress. As D/w increases, the Kt effect of the hole decreases; therefore, the allowable stress approaches the yield or ultimate stress of the material. As $\frac{F_{tu}}{E\varepsilon_u}$ decreases, the material becomes more ductile; therefore, the allowable stress approaches the yield or ultimate stress of the material.

Using this constant, allowable lug net-section tensile ultimate stress ($F_{nu}$) is calculated using the equation

$$F_{nu} = K_n F_{tu} \quad [7]$$

Similarly, allowable lug net-section tensile yield stress ($F_{ny}$) is calculated as
Based on the strength of materials equation for average normal stress in a member under axial loading, the allowable net section ultimate load (\(P_{nu}\)) is

\[
P_{nu} = F_{ny} \cdot (w - D) \cdot t \quad \text{when } F_{tu} \leq 1.304*F_{ty} \quad [9]
\]
\[
P_{nu} = 1.304 \cdot F_{ny} \cdot (w - D) \cdot T \quad \text{when } F_{tu} > 1.304*F_{ty} \quad [10]
\]

### 2.1.3 Allowable Design Load for Lug under Uniform Axial Load

The allowable design load for a lug or link under uniform axial loading is the smaller of loads obtained from equations 5 and 9 or 6 and 10. [1]

### 2.1.4 Bushing Bearing Strength under Uniform Axial Load

The allowable yield stress for bushings (\(F_{by}\)) is limited by the compressive yield strength of the bushing material (\(F_{cy,B}\)). Assuming that the bushing extends the thickness of the lug, the allowable bearing ultimate stress for the bushing (\(F_{bru,B}\)) is

\[
F_{bru,B} = 1.304 \cdot F_{cy,B} \quad [11]
\]

Based on the calculation of bearing stress in a plate, the allowable bushing ultimate load (\(P_{u,B}\)) is

\[
P_{u,B} = 1.304 \cdot F_{cy,B} \cdot D \cdot t \quad [12]
\]

### 2.1.5 Allowable Design Load for Lug/Bushing Combination under Uniform Axial Load

The allowable lug/bushing ultimate load (\(P_{u,LB}\)) is the lower of the loads obtained from section 2.1.3 and equation 12. For a double shear joint, this value is the ultimate load for the link and its bushing while twice this value is the ultimate load for the lug and its bushing. The lower of these values is the ultimate load value for the double shear joint.

### 2.1.6 Pin Shear Strength for Double Shear Joints under Uniform Axial Load

Based on strength of materials average shear stress calculations for a section, the pin ultimate shear load (\(P_{us,p}\)) is

\[
P_{us,p} = \frac{\pi}{2} \cdot D_p^2 \cdot F_{su,p} \quad [13]
\]
2.1.7 Pin Bending Strength for Double Shear Joints under Uniform Axial Load

While pin bending failure is infrequent, excessive pin deflection causes load to peak near the lug shear planes instead of being uniformly distributed across the lug thickness, leading to possible premature failure of the joint. [3] However, this concentration of load can decrease the bending arm, and therefore bending moment, on the pin. Thus, for the doubler shear joint shown in Figure 5, one must complete the following procedure to determine the true pin bending load. [1]

\[ M_{\text{max, } p} = \frac{P}{2} \left( \frac{t_1}{2} + \frac{t_2}{4} + g \right) \]  

[14]

Based on strength of materials beam bending calculations, the ultimate failing moment of the pin is

\[ M_{u, p} = \frac{\pi}{32} \cdot k_{bp} \cdot D_p^3 \cdot F_{u, p} \]  

[15]

where \( k_{bp} \) is the plastic bending coefficient for the pin. This coefficient is 1.0 for perfectly elastic materials, 1.7 for perfectly plastic materials, and 1.56 for reasonably ductile materials. Finally, the pin ultimate bending load (\( P_{ub, p} \)) can be calculated as
If $P_{ub,p}$ is greater than or equal to either $P_{us,p}$ or the values determined in section 2.1.5, then the pin is relatively strong and bending of the pin is not critical. No further pin calculations are required. Otherwise, the pin is considered relatively weak and critical in bending. Further calculations must be completed to determine a balanced design ultimate pin bending load ($P_{ub,p,max}$) that takes into account the possibility that pin loads are not uniformly distributed across the lug and link faces. [1]

The maximum allowable value of pin bending load is obtained when the widths over which the pin loads are distributed are sufficiently reduced such that the pin bending load equals the allowable lug/bushing ultimate load. This balanced design ultimate pin bending load is calculated from the equation

$$P_{ub,p,max} = 2C \sqrt{\left(\frac{P_{ub,p}}{C}\right) \cdot \left(\frac{t_1}{2} + \frac{t_2}{4} + g\right) + g^2 - 2Cg}$$  \hspace{1cm} [17]$$

Where

$$C = \frac{P_{u,LB,log} \cdot P_{u,LB,link}}{P_{u,LB,log} \cdot t_{link} + P_{u,LB,link} \cdot t_{log}}$$  \hspace{1cm} [18]$$

The balanced design effective bearing widths are

$$b_{1,min} = \frac{P_{ub,p,max} \cdot t_{log}}{2 \cdot P_{u,LB,log}}$$  \hspace{1cm} [19]$$

$$2b_{2,min} = \frac{P_{ub,p,max} \cdot t_{link}}{2 \cdot P_{u,LB,link}}$$  \hspace{1cm} [20]$$

These effective bearing widths can be used to calculate balanced design lug ultimate bearing and net-section loads as well as ultimate bushing bearing loads.
2.1.8 Lug and Link Tang Strength for Double Shear Joints under Uniform Axial Load

If the balanced design approach is not used, the load in the lugs and tangs are assumed to be uniformly distributed. The allowable stress in the tangs is $F_{u,T}$ and the lug/link tang strength is the lower of the following two values.

\[ P_T = 2F_{u,T} \cdot W_{T,lag} \cdot t_{lag} \]  \[ 21 \]

\[ P_T = F_{u,T,link} \cdot W_{T,link} \cdot t_{link} \]  \[ 22 \]

However, if the balanced design approach has been used, the lug tangs must be checked for the combined axial and bending stresses resulting from the eccentric application of bearing loads. Then, the lug tang strength is the lower of the following values

\[ P_T = \frac{2F_{u,T,lag} \cdot W_{T,lag} \cdot t_{lag}}{1 + \frac{k_{bT} \cdot \left( 1 - \frac{b_{lag,\min}}{t_{lag}} \right)}{3}} \]  \[ 23 \]

\[ P_T = F_{u,T,link} \cdot W_{T,link} \cdot t_{link} \]  \[ 24 \]

where $k_{bT}$ is the plastic bending coefficient for a lug tang of rectangular cross section. It varies from 1.0 for a perfectly elastic tang to 1.5 for a perfectly plastic tang, with 1.4 representing a material with reasonably ductility. [1]

2.1.9 Allowable Joint Ultimate Load

If the pin is relatively strong the allowable joint ultimate load ($P_{all}$) is the minimum of $P_{us,p}$ and $P_{u,LB}$. If the pin is relatively weak, $P_{all}$ is the minimum of $P_{us,p}$ and $P_{ub,p,max}$.

2.2 Lug Strength under Transverse Load

Axial loading was understood and an analysis process which provided reasonably accurate results developed long before a satisfactory method for transverse loading was determined. Initially, transversely loaded lugs were treated as redundant elastic frames uniformly loaded by a pin; however, this method was laborious and unrealistic. When stresses are in the elastic range, the load distribution is not uniform since the comparatively rigid pin prevents bending deformation in the lug and yielding of the lug further alters the stress distribution. Next, a plastic method was developed to remove the
limitations and deficiencies of the prior elastic method, but the uncertainty and laborious nature of the analysis remained. Thus, an empirical method was developed by Melcon and Hoblit by testing aluminum and steel samples loaded transversely and obliquely for both yield and ultimate strength. From this data, failure load was found to plot against a single parameter, described below in section 2.2.1. Since the 1950’s, test data has been obtained for other materials. [3]

As a transversely loaded lug, shown in Figure 6 below, is a more redundant structure than an axially loaded lug, it has a more complicated failure load and additional calculations must be completed than were done for the axially loaded lug. The strength calculations are the same as noted above; however, the maximum lug bearing stresses at ultimate and yield load must not exceed the values explained below in equations 26 and 27.

![Figure 6: Lug Geometry for Transversely Loaded Lug][1]

2.2.1 Lug Strength under Transverse Load

Similar to K in section 2.1.1, the transverse ultimate load coefficient (K_{tru}) and the transverse yield load coefficient (K_{try}) must first be determined as a function of the effective edge distance (h_{av}) using Figure 7.

\[
h_{av} = \frac{6}{\frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} + \frac{1}{h_4}} \quad [25]
\]

The averaging is on a reciprocal basis to give reasonable results when one dimension is much larger than the others. The coefficient 3 in equation 25 was determined to give the
least scatter of test data on which this method was based. Additionally, the $h_1$ dimension is at the root of the cantilever portion of the lug and therefore carries most of the load. [3]

![Figure 7: Transverse Ultimate and Yield Load Coefficients][1]

From these coefficients the lug ultimate bearing stress ($F_{bru}$) and lug yield bearing stress ($F_{bry}$) can be calculated

$$F_{bru} = K_{bru} \cdot F_{tu} \quad [26]$$

$$F_{bry} = K_{bry} \cdot F_{ty} \quad [27]$$

Based on strength of materials plate bearing stress calculations, the allowable lug transverse ultimate load ($P_{tru}$) is

$$P_{tru} = F_{bru} \cdot D \cdot t \quad \text{when } F_{tu} \leq 1.304 \cdot F_{ty} \quad [28]$$

$$P_{tru} = 1.304 \cdot F_{bry} \cdot D \cdot t \quad \text{when } F_{tu} > 1.304 \cdot F_{ty} \quad [29]$$
3. Application of Strength of Materials Models to a Simple Double Shear Joint

A simple double shear joint, show in Figure 8, was used for this analysis.

Figure 8: Sample Double Shear Joint

Its critical geometry and associated material properties are summarized in the below Table 1. All material properties were obtained from Military Handbook 5 – Metallic Materials and Elements for Aerospace Vehicle Structures (MIL-HDBK-5). [4]
### Table 1: Joint Geometry and Material Properties [4]

<table>
<thead>
<tr>
<th>Material</th>
<th>Units</th>
<th>Lug</th>
<th>Link</th>
<th>Bushing</th>
<th>Pin</th>
</tr>
</thead>
<tbody>
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<td>1000</td>
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<td>1000</td>
</tr>
<tr>
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<td>0.185</td>
<td>0.185</td>
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<tr>
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<td></td>
</tr>
<tr>
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<td>0.350</td>
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<td></td>
</tr>
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</tr>
<tr>
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<td>0.500</td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
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<tr>
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<tr>
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<tr>
<td>$h_4$</td>
<td>in</td>
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</table>

### 3.1 Lug, Bushing, and Pin Strength under Uniform Axial Loading

Based on the equations presented in Chapter 2, the above information is used to determine the critical loads for the lug, link, lug bushing, link bushing, and pin. Section 3.1.1 through 3.1.6 explain the analysis with all limiting loads summarized in Table 2 and Table 3.
3.1.1 Lug and Link Bearing Stress under Uniform Axial Load

For the lug, e/D is 0.946, suggesting a shear tear out or hoop tension failure and leading to an allowable axial load coefficient (K) of 1.635. Thus, the allowable lug ultimate bearing load (P\text{bru}) is 10,792 lb. For a double shear joint, since there are two lugs, the allowable lug ultimate bearing load (P\text{bru}) is 21,584 lb. When the effective lug bearing width is accounted for (see section 3.1.5 for calculation of this width), the allowable load for one lug decreases to 1,487 lb while the load for two lugs decreases to 2,974 lb.

For the link, e/D is 0.946, suggesting a shear tear out or hoop tension failure and leading to an allowable axial load coefficient (K) of 1.635. Thus, the allowable lug ultimate bearing load (P\text{bru}) is 8,883 lb. When the effective bearing width is accounted for (see section 3.1.5 for calculation of this width), the allowable load for the link decreases to 2,447 lb.

3.1.2 Lug and Link Net-Section under Uniform Axial Load

For the lug, D/w is 0.529, F\text{ty}/F\text{tu} is 0.838, and F\text{tu}/E\!\varepsilon\!\text{u} is 0.030, leading to a net tension stress coefficient (K\text{n}) of 0.8712. Thus, the allowable lug net section ultimate load (P\text{nu}) is 11,500 lb. For a double shear joint, since there are two lugs, the allowable lug ultimate net-section load (P\text{nu}) is 23,000 lb. When the effective lug bearing width is accounted for (see section 3.1.5 for calculation of this width), the allowable load for one lug decreases to 1,584 lb while the load for two lugs decreases to 3,168 lb.

For the link, D/w is 0.529, F\text{ty}/F\text{tu} is 0.687, and F\text{tu}/E\!\varepsilon\!\text{u} is 0.026, leading to a net tension stress coefficient (K\text{n}) of 0.8718. Thus, the allowable lug net-section ultimate load (P\text{nu}) is 9,473 lb. When the effective lug bearing width is accounted for (see section 3.1.5 for calculation of this width), the allowable load for the link decreases to 2,610 lb.

3.1.3 Lug and Link Bushing Bearing Strength under Uniform Axial Load

Limited by the compressive yield strength of the bushing material, the allowable bushing ultimate load (P\text{u,B}) for the lug bushing is 5,924 lb. For a double shear joint, since there are two lugs, the allowable bushing ultimate load (P\text{u,B}) is 11,848 lb. When the effective
lug bearing width is accounted for (see section 3.1.5 for calculation of this width), the allowable load for one lug decreases to 816 lb while the load for two lugs decreases to 1,632 lb.

The allowable bushing ultimate load \( (P_{u,B}) \) for the link bushing is 5,924 lb. When the effective lug bearing width is accounted for (see section 3.1.5 for calculation of this width), the allowable load for the link bushing decreases to 1,632 lb.

### 3.1.4 Pin Shear Strength for Double Shear Joints under Uniform Axial Load

The pin ultimate shear load \( (P_{u,p}) \) is 2,835 lb. This value is not affected by the balanced design approach.

### 3.1.5 Pin Bending Strength for Double Shear Joints under Uniform Axial Load

Assuming a uniformly distributed load along the lugs and link, the pin ultimate bending load \( (P_{u,p}) \) is 317 lb. As this value is less than the pin ultimate shear load and all limiting loads for the lug, link, and their respective bushings, the pin is weak and the critical component of the double shear joint. Thus, the balanced design approach must be undertaken. The balanced pin ultimate bending load \( (P_{ub,p,max}) \) is 1,632 lb. The balanced design effective bearing widths \( (b_{1,min} \text{ and } 2b_{2,min}) \) are 0.069 in (13.8% of the total width) and 0.138 (27.5% of the total width) respectively. Thus, the allowable joint ultimate load \( (P_{all}) \) is 1,632 lb and the joint is limited by the pin and the bushings.

### 3.1.6 Lug and Link Tang Strength for Double Shear Joints under Uniform Axial Load

Since the balanced design method is used for this configuration, the lug tangs are checked for the combined axial and bending stresses resulting from the eccentric application of bearing loads. The lug tang strength is 19,665 lb while the link tang strength is 25,725 lb.
3.2 Lug Strength under Transverse Load

Based on the equations presented in Chapter 2, the above information is used to determine the critical loads for the lug. Section 3.2 explains the analysis with the limiting loads summarized in Table 2.

Since the ultimate tensile strength of the lug material ($F_{tu}$) is less than 1.304 times the tensile yield strength of the lug material ($F_{ty}$), the transverse ultimate load coefficient ($K_{tru}$) is determined to be 0.6751. Thus, the allowable lug transverse ultimate load ($P_{tru}$) for one lug is 9,992 lb. For a double shear joint, allowable lug transverse ultimate load ($P_{tru}$) for two lugs is 19,984 lb. When the effective lug bearing width is accounted for (see section 3.1.5 for calculation of this width), the allowable load for one lug decreases to 1,376 lb while the load for two lugs decreases to 2,753 lb.

Table 2: Limiting Loads for Lug, Link, Bushings

<table>
<thead>
<tr>
<th></th>
<th>Limiting Load Lug</th>
<th>Limiting Load Link</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{bru}$ (lb)</td>
<td>2,974</td>
<td>2,447</td>
</tr>
<tr>
<td>$P_{nu}$ (lb)</td>
<td>3,168</td>
<td>2,610</td>
</tr>
<tr>
<td>$P_{u,B}$ (lb)</td>
<td>1,632</td>
<td>1,632</td>
</tr>
<tr>
<td>$P_{T}$ (lb)</td>
<td>19,665</td>
<td>25,725</td>
</tr>
<tr>
<td>$P_{tru}$ (lb)</td>
<td>2,753</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3: Limiting Loads for Pin

<table>
<thead>
<tr>
<th></th>
<th>Limiting Load Pin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{us,p}$ (lb)</td>
<td>2,835</td>
</tr>
<tr>
<td>$P_{ub,p,max}$ (lb)</td>
<td>1,632</td>
</tr>
</tbody>
</table>
4. Lug Analysis Using the Finite Element Method and Comparison with Strength of Materials Model Calculations

4.1 Description of ANSYS Model

A symmetric, parametric, solid model of the double shear joint was created in ANSYS using the geometry outlined in Chapter 3. Symmetry was taken into account to minimize the model size and allow for finer meshes in required regions. While a quarter symmetry model could have been used for the axially loaded case, a half symmetry model was used for both the axially and transversely loaded models such that a simplified macro could be used to create them both. Joint geometry, material properties, plasticity assumptions, mesh densities, and element types were all parameterized to allow for easy variation of the joint, rapid turn around of models, and consistent modeling techniques. The model, colored by material type, is depicted below in Figure 9. As this is a half model, the symmetry plane is located on the face of the link that can not be seen in the picture.

![Figure 9: ANSYS Model of Doubler Shear Joint](image)

The model was meshed with SOLID95 elements, which are quadratic, brick elements. The mesh was locally refined in the regions of expected plasticity and high load. The lug and link were divided such that regions radially closer to the pin have a finer mesh than regions further away. Additionally, as excessive pin deflection causes load to peak near the lug shear planes, the mesh in the lug and link in this region was more refined.
than in regions relatively far away from the shear plane. Figure 10 shows basic mesh for
the joint. A mesh study was completed to determine the appropriate mesh for all
components and will be described in section 4.2.

![Figure 10: Sample Mesh of Doubler Shear Joint](image)

Surface to surface contact (CONTACT174 and TAREGT170) was used between the
joint components. Standard contact with default contact settings was used between the
pin and the bushings. As the theoretical hand calculations do not take into account an
interference fit between the bushing and lug/link, those components were modeled as
line on line with standard contact between them. Additionally, the model was created
with an aligned mesh between the bushing and lug/link. Initially, a coefficient of
friction of 0.3 was used for all joints; however, this made the joint too stiff and was thus
decreased to 0.01.

Symmetry boundary conditions were applied to the link and pin. The lug was grounded
in the normal and shear direction at its base. To prevent the pin from spinning about its
axis, a mass element (MASS21) was created at the center of the pin on the symmetry
plane. This mass element was constrained about the pin’s axis and connected to the pin
elements on the symmetry plane with rigid constraint equations in all directions, with the
mass element as the master node. The contact between the bushings and the pin
prevented the bushings from spinning in the joint. The load was applied to one node on
the link face. Nodes on the link cut face were coupled normal to the face to provide a moment constraint and to distribute this force over the face. The same nodes were also connected via constraint equations to evenly distribute the shear force over the face. These constraints are depicted in Figure 11.

![Figure 11: Double Shear Joint Constraints and Load Application](image)

Plasticity was initially accounted for in the material properties of the lug, link, and pin as these components are allowed to yield. True stress-strain curves for a single temperature were created based on data in Military Handbook 5. [4] The proportional limit is assumed to be 85% of the yield strength, which, along with ultimate strength and ultimate strain, are input by the analyst. All models are solved non-linearly with large deflection on since true stress-strain curves are used. Thus, the ANSYS predicted stresses will be true stresses and will be converted to engineering stress before stress margin is calculated using the equation:

$$\sigma_{true} = \left(1 + \varepsilon_{engineering}\right) \sigma_{engineering}$$

[30]

4.2 Mesh Density Study

A mesh density study was undertaken on the purely axially loaded joint to ensure that the FE model was predicting stress and strain correctly. Initially, the coarse mesh seen
in Figure 10 and Figure 11 was used to debug the model and ensure that it solved. This mesh used six elements along the length of the lug and five elements along the length of the link with two elements in the refined regions, which were 15% of the link and 25% of the lug, near the lug and link shear planes. Thus, the average element length was 0.094” in the coarse region of the lug and 0.063” in the refined region. Additionally, the average element length was 0.071” in the coarse region of the link and 0.019” in the refined region. These refined regions encompassed the predicted balanced design bearing widths calculated in section 3.1.5. There were 16 elements around the lug and link diameters and 24 elements around the pin diameter, making the average element length 0.036” and 0.018” respectively. Note that these values are a fraction of the average lug and link element lengths.

As was predicted, although the model ran quickly, stress and strains were not converged and the contact regions between the pin and bushings were poorly defined. Error in predicted stress was calculated using the error estimation tool (ERNORM command) in ANSYS. It provided the predicted stress value as well as the max bound that the stress could reach. Thus, the error was calculated from the difference in these values divided by the max possible value. Error for the lug, link, lug bushing, link bushing, and pin were 7%, 27%, 56%, 6%, and 15%, respectively, based on von Mises stresses from which ANSYS determines whether the part has yielded.

The mesh was refined such that predicted stress error was less than 10%. The length of the refined region was modified such that 25% of both the lug and the link had a refined mesh. Both the lug and the link were meshed such that, if the entire length was meshed with the same element size, there would be 12 elements along the length. In the refined region of both, there were four elements. Thus, for the lug, the average element length in the coarse region was 0.040” and the average element length in the refined region was 0.025”. Additionally, for the link, the average element length in the coarse region was 0.0375” and the average element length in the refined region was 0.0156”. There were 24 elements around the diameters of all components, such that the average element length around the lug and link diameter and bushing outer diameter was 0.024” while the
average element length around the bushing inner diameter and pin diameter was 0.018”.
This mesh refinement also ensured that the pin, bushing, and lug/link meshes were
aligned in both the radial and axial direction, which led to stress and contact pressure
convergence. The final mesh can be seen in Figure 12.

![Refined Mesh for Double Shear Joint](image)

Figure 12: Refined Mesh for Double Shear Joint
Run time increased by a factor of three. This refinement altered the predicted deflection
of the joint by less than 1%. Stress percent error for the lug, link, lug bushing, link
bushing, and pin were reduced to 6%, 7%, 8%, 8%, and 1% respectively. The largest
area of uncertainty existed in the link near the shear plane and in the pin at this same
location.

4.3 Pin Plasticity Study

Care was taken to ensure that the ANSYS model treated the joint components the same
way that the theoretical calculation did. The most important decision was for which
components plasticity should be considered. Initially, the lug, link, and pin were
modeled to allow for plasticity through the inclusion of true stress-strain curves. The
bushings were treated elastically as they were designed to not yield in compression.
However, this method was reconsidered after the initial mesh studies were completed.
Since the theoretical hand calculations use a plastic bending coefficient to augment the
ultimate tensile allowables, as can be seen in equation 15 of section 2.1.7, a study was
undertaken in which the refined model outlined above was run (1) treating the lug, link,
and pin plastically and (2) treating the pin and bushings elastically and the link and lug plastically. For each case, the limiting axial load of 1,632 lb. was applied.

While peak stresses and predicted loads were similar between the two cases, a significant difference existed in the distance over which contact was predicted between the pin and bushing. This value should correlate with the effective width for both the lug and link of 0.069” calculated in section 3.1.5. Contact regions were defined based on contact pressure between the pin and the bushing. The die out in this pressure was plotted and overall contact distance was determined based on varying minimum pressures.

For Case 1, in which the pin was treated plastically, the peak contact pressure under the lug was 145 ksi while the peak pressure under the link was 165 ksi. The subsequent contact pressure die out for regions of the pin under the lug and link can be seen in Figure 13. A contact pressure plot from ANSYS is shown with the stress die out directions depicted. Since the model is a half model, the symmetry plane at the center of the pin and link is shown. Thus, there will be contact between the pin and link bushing on the mirror image side of the link as well. Additionally, an EXCEL plot is provided to better understand the distance over which the pressure die out takes place.
For the lug region, it was determined that if contact was defined as the distance between the peak pressure and 10% of the peak pressure, then the contact length was 0.115". However, if contact was defined as the distance between the peak pressure and 25% of the peak pressure, then the contact length was 0.081". To match the theoretical effective width prediction of 0.069", contact would have to be defined as the distance between the peak pressure and 31% of the peak pressure. Note that a small region of contact also occurs between the pin and the bushing at the end of the lug, although the peak contact pressure value in this region is 2% of the overall peak stress. For the link region, it was determined that if contact was defined as the distance between the peak pressure and 10% of the peak pressure, then the contact length was 0.136". However, if contact was defined as the distance between the peak pressure and 25% of the peak pressure, then the contact length was 0.071". To match the theoretical effective width prediction of 0.069", contact would have to be defined as the distance between the peak pressure and 26% of the peak pressure.
For Case 2, in which the pin was treated elastically, the peak contact pressure under the lug was 184 ksi while the peak pressure under the link was 251 ksi. The subsequent contact pressure die out for regions of the pin under the lug and link can be seen in Figure 14. A contact pressure plot from ANSYS is shown with the stress die out directions depicted. Since the model is a half model, the symmetry plane at the center of the pin and link is shown. Thus, there will be contact on the mirror image side of the link as well. Additionally, an EXCEL plot is provided to better understand the distance over which the pressure die out takes place.

For the lug region, it was determined that if contact was defined as the distance between the peak pressure and 10% of the peak pressure, then the contact length was 0.097”. However, if contact was defined as the distance between the peak pressure and 15% of the peak pressure, then the contact length was 0.085”. To match the theoretical effective width prediction of 0.069”, contact would have to be defined as the distance between the peak pressure and 21% of the peak pressure. Note that a small region of contact also occurs between the pin and the bushing at the end of the lug, although the peak contact
pressure value in this region is 2% of the overall peak stress. For the link region, it was determined that if contact was defined as the distance between the peak pressure and 10% of the peak pressure, then the contact length was 0.086”. However, if contact was defined as the distance between the peak pressure and 15% of the peak pressure, then the contact length was 0.070”. To match the theoretical effective width prediction of 0.069”, contact would have to be defined as the distance between the peak pressure and 16% of the peak pressure.

Comparing the two cases, it can be seen that Case 2 better represents the contact between the joint components. A greater percentage of the ANSYS contact zone can be considered while still ensuring correlation between the ANSYS model and the theoretical calculations. Thus, going forward, the pin will be treated elastically and contact will be defined as the distance between the peak pressure and 15% of the peak pressure. This assumption results in an ANSYS predicted effective width of 0.085” for the pin and 0.070” for the link. Note that since the model is symmetric, the overall contact width for the link is 0.140”.

4.4 Comparison of Strength of Materials Calculations to FEA Analysis of Lug, Bushing, and Pin under Uniform Axial Loading

After the mesh density and pin plasticity studies were completed, ANSYS stress margin was calculated using two methods, peak stress and stress averaged over the contact area. These margins were compared to the margins predicted by the theoretical calculations.

4.4.1 Lug and Link Bearing Stress under Uniform Axial Load

In section 3.1.1, it was calculated that the ultimate bearing load for one lug is 1,487 lb while the load for two joint lugs is 2,974 lb. Since the limiting joint load is 1,632 lb, the predicted margin of safety is 0.82.

As c/D for the lug is less than 1.5, it is predicted that the lug will fail in shear tear out or hoop tension. Therefore, both von Mises and directional stresses were analyzed when peak stress was considered. ANSYS used von Mises stress to determine when a part has
exceeded its proportional limit. One can also see where a part has gone plastic by plotting the stress state ratio and noting where the ratio is greater than 1.0. For low amounts of plasticity, as was expected in this model, the von Mises stress should agree with the plastic equivalent stress. For the lug, von Mises stress and the stress state ratio at the lug shear plane can be seen in Figure 15.

![Figure 15: Von Mises Stress and Stress State Ratio for the Lug](image)

Directional stress seen in Figure 16 was used to calculate peak hoop tension stress margin.

![Figure 16: Stress in the Y Direction for the Lug](image)

The bearing ultimate stress allowable, calculated based on the material properties presented in Chapter 3 and equation 1 in section 2.1.1, is 117 ksi. As the peak stress at the top of the lug hole is -85 ksi, the calculated margin of safety is 0.38, less than the theoretical value. Thus, using peak stress is conservative.

To determine the average stress for the lug, loads were extracted from the ANSYS model at both the hoop tension plane and the shear/bearing plane, which was 40° from
the top of the lug. Extracted loads were 444 lb for the hoop tension plane and 457 for the shear/bearing plane. Area was calculated based on the effective width determined in the previous section. For both planes, area is 0.007 in². Thus, the calculated stresses are 63 ksi and 65 ksi, respectively, which result in calculated margins of safety of 0.84 and 0.79. These margins bracket the predicted margin of 0.82 and differ from the theoretical margin by approximately 2%.

In section 3.1.1, it was calculated that the ultimate bearing load for the link is 2,447 lb. Since the limiting joint load is 1,632 lb, the predicted margin of safety is 0.50.

As e/D for the link is less than 1.5, it is predicted that the link will fail in shear tear out or hoop tension. Therefore, both von Mises and directional stresses were analyzed when peak stress was considered. For the link, von Mises stress and the stress state ratio at the link shear plane can be seen in Figure 17.

![Figure 17: Von Mises Stress and Stress State Ratio for the Link](image)

Directional stress seen in Figure 18 was used to calculate peak hoop tension stress margin.
Figure 18: Stress in the Y Direction for the Link

The bearing ultimate stress allowable, calculated based on the material properties presented in Chapter 3 and equations 3 and 6 in section 2.1.1, is 96 ksi. As the peak stress at the top of the link hole is -85 ksi, the calculated margin of safety is 0.14, less than the theoretical value. Thus, using peak stress is conservative.

To determine the average stress for the link, loads were extracted from the ANSYS model at both the hoop tension plane and the shear/bearing plane, which was 40° from the top of the lug. Area was calculated based on the effective width determined in the previous section. For both planes, area is 0.012 in². Thus, the calculated margins of safety were 0.67 and 0.72. These margins differ for the theoretical value by approximately 20% and are slightly anticonservative. Thus, the peak margin and the net section margin bracket the margin predicted by the theoretical calculations.

4.4.2 Lug and Link Net-Section under Uniform Axial Load

In section 3.1.2, it was calculated that the ultimate net-section load for one lug is 1,584 lb while the load for two joint lugs is 3,168 lb. Since the limiting joint load is 1,632 lb, the predicted margin of safety is 0.94.

Directional stress seen in Figure 16 was used to calculate peak net-section stress margin. The net-section ultimate stress allowable, calculated based on the material properties presented in Chapter 3 and equation 7 in section 2.1.2, is 139 ksi. As the peak stress at
the net-section plane of the lug hole is 117 ksi, the calculated margin of safety is 0.19, less than the theoretical value. Thus, using peak stress is conservative.

To determine the average stress for the lug, loads were extracted from the ANSYS model at the net-section plane, shown in Figure 19.

![Figure 19: Lug Net-Section Plane from which Peak and Average Stress were Calculated](image)

The extracted load was 982 lb. Area was calculated based on the effective width determined in previous section and was 0.014 in². Thus, the calculated stress was 70 ksi, which resulted in a calculated margin of safety of 0.99. This margin varies from the predicted margin by approximately 5%.

In section 3.1.2, it was calculated that the ultimate net-section load for the link is 2,610 lb. Since the limiting joint load is 1,632 lb, the predicted margin of safety is 0.60.

Directional stress seen in Figure 16 Figure 18 was used to calculate peak net-section stress margin. The net-section ultimate stress allowable, calculated based on the material properties presented in Chapter 3 and equations 8 and 10 in section 2.1.2, is 115 ksi. As the peak stress at the net-section plane of the link hole was 92 ksi, the calculated margin of safety was 0.25, less than the theoretical value. Thus, using peak stress is conservative.
To determine average stress for the link, loads were extracted from the ANSYS model at the net-section plane, shown in Figure 20.

![Figure 20: Link Net-Section Plane from which Peak and Average Stress were Calculated](image)

The extracted load was 1719 lb. Area was calculated based on the effective width determined in previous section and was 0.023 in². Thus, the calculated stress was 74 ksi, which resulted in a calculated margin of safety of 0.54. This margin varies from the predicted margin by approximately 10%.

For both the lug and the link, average stress provides a better correlation with the theoretical calculations than does peak stress. For the lug, the margins calculated from these two approaches bracket the theoretical margin while for the link, the ANSYS model is more conservative.

### 4.4.3 Lug and Link Bushing Bearing Strength under Uniform Axial Load

In section 3.1.2, it was calculated that the allowable bushing ultimate load for one lug bushing is 816 lb while the load for two lug bushings in the joint is 1,632 lb. Since the limiting joint load is 1,632 lb, the predicted margin of safety is 0.00.
Figure 21 shows the radial stress of the lug bushing at the shear plane.

![Figure 21: Lug Bushing Radial Stress](image)

The lug bushing ultimate compressive stress allowable, calculated based on the material properties presented in Chapter 3 and equation 11 in section 2.1.4, is 88 ksi. As the peak stress at the top of the lug bushing is -207 ksi, the calculated margin of safety is -0.45, less than the theoretical value. Thus, using peak stress is conservative.

Since the region where the bushing was overstressed was very local, an average stress was calculated based on the previously determined effective width, 0.085”, and the top 1/6th of the bushing. This selected region can be seen in Figure 22. The average radial stress in this region was -80 ksi. Thus, the associated margin of safety was 0.09. As was seen before, the peak stress margin and the average stress margin bracket the margin predicted by the theoretical calculations; however, the average stress margin is much closer.
In section 3.1.2, it was calculated that the allowable bushing ultimate load for the link bushing is 1,632 lb. Since the limiting joint load is 1,632 lb, the predicted margin of safety is 0.00.

Figure 23 shows the radial stress of the link bushing at the shear plane.
The link bushing ultimate compressive stress allowable, calculated based on the material properties presented in Chapter 3 and equation 11 in Section 2.1.4, is 88 ksi. As the peak stress at the top of the link bushing was -250 ksi, the calculated margin of safety was -0.54, less than the theoretical value. Thus, using peak stress is conservative.

Since the region where the bushing is overstressed was very local, an average stress was calculated based on the effective width previously determined, 0.070”, and the top 1/6th of the bushing. This selected region can be seen in Figure 24. The average radial stress in this region was -84 ksi. Thus, the associated margin of safety was 0.04. As was seen before, the peak stress margin and the average stress margin bracket the margin predicted by the theoretical calculations; however, the average stress margin is much closer.

Figure 24: Region of Link Bushing used for Average Stress Calculation
4.4.4 Pin Shear Strength for Double Shear Joints under Uniform Axial Load

In section 3.1.4, it was calculated that the allowable shear load for the pin is 2,834 lb. Since the limiting joint load is 1,632 lb, the predicted margin of safety is 0.74.

Pin peak shear stress occurs at the shear planes of the lug and link as can be seen in Figure 25. This figure is of half the pin with the symmetry plane in the lower right hand corner of the plot.

![Figure 25: Shear Stresses in the Pin](image)

The ultimate shear allowable is 99 ksi. As the peak stress at the top of the link bushing was -178 ksi, the calculated margin of safety was -0.44, less than the theoretical value. Thus, using peak stress is conservative.

To determine average shear, loads were extracted from the ANSYS model at the link shear plane. The shear load on this face of the pin was 786 lb. Thus, the calculated shear stress was 55.6 ksi and the margin of safety was 0.78. While this margin is slightly anticonservative based on the theoretical value, it differs by only 5%.

4.4.5 Pin Bending Strength for Double Shear Joints under Uniform Axial Load

In section 3.1.5, it was calculated that the allowable bending load for the pin is 1,632 lb. Since the limiting joint load is 1,632 lb, the predicted margin of safety is 0.00.

Pin peak bending stress occurs at the ends of the lug and link effective widths as can be seen in Figure 26. This figure is of half the pin with the symmetry plane in the lower right hand corner of the plot. Note that the highest stresses occur under the link.
The pin bending ultimate stress allowable, calculated based on the material properties presented in Chapter 3 and equation 16 in section 2.1.7, is 250 ksi. The bending stress under the link was 104 ksi. Thus, the calculated margin of safety was 1.40, much greater than the theoretical value. The bending stress under the lug was 99 ksi. Thus, the calculated margin of safety was 1.52, also much greater than the theoretical value.

This discrepancy could be attributed to pin to bushing contact. The pin contact plot in Figure 14 shows that additional contact occurs at the end of the pin between the pin and the lug bushing. This differs from the theoretical pressure profile shown in Figure 5. The theoretical profile has contact between the pin and bushing and/or lug only near the lug/link shear plane. The additional contact in the ANSYS model restricts the pin from bending and decreases the load supported by each region of contact. Thus, the bending moment and, subsequently, bending stresses decrease.

4.5 Comparison of Strength of Materials Calculations to FEA Analysis of Lug under Uniform Transverse Loading

After the axially loaded joint studies were completed, a transversally loaded joint was analyzed. With the link rotated 90°, this joint had the same mesh density and applied load as the axially loaded joint. The joint geometry, mesh, and loading can be seen in Figure 27.
Stress margin was calculated using two methods, peak stress and stress averaged over the contact area. These margins were compared to the margins predicted by the theoretical calculations.

### 4.5.1 Lug Strength under Transverse Load

In section 3.2, it was calculated that the allowable ultimate transverse load for one lug is 1,376 lb while the load for two lugs is 2,753 lb. Since the limiting joint load is 1,632 lb, the predicted margin of safety is 0.69.

The lug ultimate bearing stress allowable, calculated based on the material properties presented in Chapter 3 and equation 26 in section 2.2.1, is 108 ksi. As the peak stress due to bearing between the lug bushing and the lug was -91 ksi, and can be seen in Figure 28, the calculated margin of safety was 0.19, less than the theoretical value. Thus, using peak stress is conservative.
Figure 28: Transverse Stress in the Lug

As was done in the pin plasticity study in section 4.3, the effective width was calculated by plotting contact pressure die out under the lug and determining the distance between the peak pressure and 15% of the peak pressure. The contact pressure die out for the region of the pin under the lug can be seen in Figure 29. A contact pressure plot from ANSYS is shown with the stress die out direction depicted. Since the model is a half model, this is the contact plot for one lug. Additionally, an EXCEL plot is provided to better understand the distance over which the pressure die out takes place. The effective length is determined to be 0.100”, a value greater than the 0.085” effective length calculated for the axially loaded joint. To match the theoretical effective with prediction of 0.069”, contact would have to be defined as the distance between the peak pressure and 25% of the peak pressure. Note that the peak contact is not centered, but is slightly below the centerline of the pin and lug.
Figure 29: Contact Pressure Die Out for Transversely Loaded Lug

Since the region of peak lug stress is very local, an average stress was calculated based on the 0.100” effective width and the surface of the lug upon which the bushing and pin bear. This region can be seen in Figure 30. The average stress in this region was -61 ksi. Thus, the associated margin of safety was 0.77, 11% greater than the margin calculated from the theoretical calculations. As was seen before, the peak stress margin and the average stress margin bracket the margin predicted by the theoretical calculations; however, the average stress margin is much closer.
Figure 30: Region of Lug used for Average Stress Calculation
5. Conclusions

Prior to the 1950’s, lugs were overdesigned as weight and space were not design driving factors. With the tightening of weight, cost, and space requirements in the aerospace industry, a theoretical lug analysis based on basic strength of materials calculations and empirical data was developed in the 1950’s and 1960’s by engineers at both Lockheed Aircraft Corporation and the United States Air Force. With the increasing prevalence and usage of finite element analysis codes, it is necessary to determine whether the results obtained from FE analysis concur with those historically acceptable values generated from theoretical hand calculations. Thus, this project was undertaken to compare these two values for a set geometry.

Mesh density and plasticity studies were undertaken to ensure that the FE model was predicting stress and strain correctly and that the ANSYS model treated the joint components the same way that the theoretical calculation did. The mesh was refined such that predicted stress error was less than 10%. One key factor in stress and contact pattern convergence was ensuring that the pin, bushing, and lug/link meshes were aligned in the pin axial direction. The plasticity study concluded that the pin should be modeled with elastic material properties and not a true stress-strain curve as the lug and link were. This assumption resulted in contact patterns that best matched those predicted by the theoretical calculations. Additionally, this study resulted in the defining of contact between the pin and the bushing as the distance between the peak pressure and 15% of the peak pressure.

Ultimately, the finite element analysis compared well with the theoretical predictions. Margin of safety was calculated based on ultimate load for the theoretical calculation. For the finite element model, margin of safety was calculated using two methods, peak stress and stress averaged over the contact area. Using peak stress was very conservative and predicted margins were much less than those calculated from the theoretical calculations. Table 4-5 summarize the predicted margin for theoretical calculations and average FE stresses.
Table 4: Comparison of Strength of Materials and FEA Lug Margin of Safety for Uniform Axial Load

<table>
<thead>
<tr>
<th></th>
<th>Theoretical MS</th>
<th>Finite Element Average Stress MS</th>
<th>Percent Difference in MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear/Bearing Stress</td>
<td>0.82</td>
<td>0.79</td>
<td>2</td>
</tr>
<tr>
<td>Hoop Tension Stress</td>
<td>0.82</td>
<td>0.84</td>
<td>2</td>
</tr>
<tr>
<td>Net-Section Stress</td>
<td>0.94</td>
<td>0.99</td>
<td>5</td>
</tr>
<tr>
<td>Bushing – Bearing Stress</td>
<td>0.00</td>
<td>0.09</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5: Comparison of Strength of Materials and FEA Link Margin of Safety for Uniform Axial Load

<table>
<thead>
<tr>
<th></th>
<th>Theoretical MS</th>
<th>Finite Element Average Stress MS</th>
<th>Percent Difference in MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear/Bearing Stress</td>
<td>0.50</td>
<td>0.67</td>
<td>20</td>
</tr>
<tr>
<td>Hoop Tension Stress</td>
<td>0.50</td>
<td>0.72</td>
<td>20</td>
</tr>
<tr>
<td>Net-Section Stress</td>
<td>0.60</td>
<td>0.54</td>
<td>10</td>
</tr>
<tr>
<td>Bushing – Bearing Stress</td>
<td>0.00</td>
<td>0.04</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6: Comparison of Strength of Materials and FEA Pin Margin of Safety for Uniform Axial Load

<table>
<thead>
<tr>
<th></th>
<th>Theoretical MS</th>
<th>Finite Element Average Stress MS</th>
<th>Percent Difference in MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Stress</td>
<td>0.74</td>
<td>0.78</td>
<td>5</td>
</tr>
<tr>
<td>Bending Stress</td>
<td>0.00</td>
<td>1.52</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7: Comparison of Strength of Materials and FEA Lug Margin of Safety for Uniform Transverse Load

<table>
<thead>
<tr>
<th></th>
<th>Theoretical MS</th>
<th>Finite Element Average Stress MS</th>
<th>Percent Difference in MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing Stress</td>
<td>0.69</td>
<td>0.77</td>
<td>11</td>
</tr>
</tbody>
</table>
Note that in most cases, the FE average stress margin of safety is less conservative than the margin predicted by the theoretical calculation; however, this difference is generally less than 10%. The main exception is bending stresses in the pin. This discrepancy could be attributed to pin to bushing contact. The pin contact plot in Figure 14 shows that additional contact occurs at the end of the pin between the pin and the lug bushing. This differs from the theoretical contact profile shown in Figure 5. The theoretical profile has contact between the pin and bushing and/or lug only near the lug/link shear plane. The additional contact in the ANSYS model restricts the pin from bending and decreases the load supported by each region of contact. Thus, the bending moment and, subsequently, bending stresses decrease.

In this analysis, the lug and link yielded, but only in small regions. Thus, it is not certain that the above described method of finite element modeling would be appropriate for a joint in which the components greatly exceeded their yield strengths and had large plastic regions. Additional study would need to be undertaken to ensure that comparing average stress margin to load margin is comparable or whether load margin should be compared to margin calculated based on strain or another entity.

Overall, the FE model compared well with the theoretical calculations and analysts should look to use FE model in future design of lug joints.
6. References


