Numerical Results for the Solution of the Graetz Problem for a Bingham Plastic in Laminar Tube Flow with Constant Wall Temperature

B. F. Blackwell

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Numerical Results for the Solution of the Graetz Problem for a Bingham Plastic in Laminar Tube Flow with Constant Wall Temperature

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ABSTRACT

The Graetz problem of developing temperature profile in a tube for a fully developed laminar velocity profile has been numerically solved for a Bingham plastic. Constant properties were assumed and viscous dissipation was ignored. Results are presented for local Nusselt number, average Nusselt number, and bulk fluid temperature each as a function of axial distance from the tube inlet. The laminar Newtonian fluid is a special case of the Bingham plastic; the results presented in this article for this case appear to be more accurate than those available in the literature.

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NOMENCLATURE

c \quad \tau_y/\tau_w, \text{ ratio of yield stress to wall shear stress}
C_p \quad \text{specific heat at constant pressure}
C_n \quad \text{constant in series solution}
D \quad \text{pipe diameter}
G_n \quad \text{constant, see Eq. (12)}
h_x \quad \text{local convective heat transfer coefficient, } \frac{h_x}{D/k}, \text{ local Nusselt number}
k \quad \text{thermal conductivity}
Nu_x \quad \text{average of } Nu_x \text{ between entrance and axial location } x
Nu_m \quad \text{wall heat flux}

\dot{q}(x) \quad \frac{\bar{u}D/r}{a}, \text{ Peclet number}
r \quad \text{radial coordinate}
r_0 \quad \text{pipe radius}
r^+ \quad = r/r_0, \text{ dimensionless radius}
R_n(r^+) \quad \text{eigenfunction}
t(x,r) \quad \text{temperature}
t_b \quad \text{bulk or mixing cup temperature}
t_e \quad \text{uniform entrance temperature}
t_0 \quad \text{uniform wall temperature}
u(r) \quad \text{axial velocity}
u_{\text{max}} \quad \text{maximum axial velocity}
\bar{u} \quad \text{average axial velocity}
u^+ \quad = u/\bar{u}
x \quad \text{axial coordinate}
x^+ \quad = \frac{x/r_0}{Pe}, \text{ dimensionless axial coordinate}
GREEK

\[ \alpha = \frac{k}{\rho c_p}, \text{ thermal diffusivity} \]

\[ \eta = \text{Bingham viscosity} \]

\[ \theta = \frac{t_o - t(x,r)}{t_o - t_e}, \text{ dimensionless temperature} \]

\[ \theta_b = \frac{t_o - t_b}{t_o - t_e}, \text{ dimensionless bulk fluid temperature, see Eq. (13)} \]

\[ \lambda = \text{eigenvalue} \]

\[ \rho = \text{density} \]

\[ \tau = \text{local shear stress} \]

\[ \tau_w = \text{wall shear stress} \]

\[ \tau_y = \text{yield shear stress} \]
Numerical Results for the Solution of the Graetz Problem for a Bingham Plastic in Laminar Tube Flow with Constant Wall Temperature

INTRODUCTION

Many fluids exhibit a yield stress, a stress which must be exceeded before the fluid will flow. Bird, et. al. [1] presented an extensive tabulation of materials with yield stresses; some common examples are drilling mud, sewage sludge, grease, paint, and thorium dioxide/methanol. If the local shear stress does not exceed the yield stress, these fluids will not support a velocity gradient. In pipe flow geometries, it is possible that the fluid region near the centerline (low shear stress, \( r < r_y \)) may move as a solid (plug flow) while the fluid near the wall (high shear stress, \( r > r_y \)) supports a velocity gradient. Figure 1 presents representative laminar velocity profiles for Bingham plastics that exhibit a plug flow region.

This article was motivated by the desire to understand the heat transfer behavior of aqueous foams being used as a drilling fluid in high temperature petroleum and geothermal formations. In some applications, aqueous foams offer several advantages over conventional drilling fluids: 1) bottom hole pressure is reduced because aqueous foams have a much lower density than conventional drilling muds, 2) relatively little fall back of cuttings when circulation stops, and 3) low loss of circulation in porous formations. Additional details on the thermal behavior of aqueous foams circulating in geothermal wellbores are presented in Blackwell and Ortega [2]. This report is an extension of the work of Wissler and Schechter [3] concerning the heat transfer behavior of Bingham plastics in developing tube flow. Slip at the wall has been ignored in this analysis.
ANALYSIS

The constitutive equation for a Bingham plastic in pipe flow is of the form [1,3,4]

\[
\frac{du}{dr} = 0 \text{ for } \tau < \tau_y
\]

\[
-\frac{du}{dr} = \frac{1}{\eta} (\tau - \tau_y) \text{ for } \tau \geq \tau_y
\]

where \( u \) is the axial velocity component, \( r \) is the radial coordinate, \( \tau \) is the local shear stress, \( \tau_y \) is the yield stress, and \( \eta \) is the Bingham viscosity. For constant properties, the fully developed velocity profile has been shown to be [1,3,4]

\[
u = \frac{\tau_w r_o}{2\eta} [1 - \left(\frac{r}{r_o}\right)^2 - 2c(1 - \frac{r}{r_o})] \quad c < \frac{r}{r_o} < 1
\]

\[
u = \frac{1}{4\eta} \left(1 - \frac{4}{3}c + \frac{2}{3}c^2\right)
\]

where \( \tau_w \) is the wall shear stress, \( r_o \) is the pipe radius, and \( c = \tau_y/\tau_w \). The maximum velocity \( u_{max} \) and the average velocity can be expressed as

\[u_{max} = \frac{\tau_w r_o}{2\eta} (1-c)^2\]

\[
\bar{u} = \frac{\tau_w r_o}{4\eta} \left(1 - \frac{4}{3}c + \frac{2}{3}c^2\right)
\]

The dimensionless form of Eq.(4) is presented in Fig. 1. Note that \( c=1 \) corresponds to plug flow (\( u = u_{max} \)) while \( c=0 \) corresponds to laminar Newtonian flow.

If axial conduction is neglected (Pe>100) and viscous dissipation ignored, the steady flow constant property form of the energy equation and its boundary conditions can be written as

\[
\rho Cp u(r) \frac{\partial t}{\partial x} = \frac{k}{r} \frac{\partial}{\partial r} \left(\frac{\rho Cp u(r) \partial t}{r}\right), \quad t(0,r) = t_e, \quad t(x,r_0) = t_o,
\]

\[
\frac{\partial t(x,0)}{\partial r} = 0
\]
This analysis is restricted to Prandtl number > 1 but still sufficiently small that viscous dissipation is not important. The following dimensionless variables will be useful:

\[
\theta = \frac{t_0 - t(x, r)}{t_0 - t_e}, \quad r^+ = \frac{r}{r_o}, \quad u^+ = \frac{u}{\bar{u}}, \quad x^+ = \frac{x/r_o}{Pe}, \quad Pe = \frac{\bar{U}D}{\alpha} \quad (6)
\]

where \( t_0 \) is the wall temperature, \( t_e \) is the uniform inlet fluid temperature, \( \alpha \) is the thermal diffusivity, and \( Pe \) is the dimensionless Peclet number. For uniform wall temperature \( t_0 \) and inlet temperature \( t_e \), the dimensionless energy equation is

\[
\frac{u^+}{2} \frac{\partial \theta}{\partial x^+} = \frac{1}{r^+} \frac{\partial}{\partial r^+} \left( r^+ \frac{\partial \theta}{\partial r^+} \right), \quad \theta(o, r^+)=1, \quad \theta(x^+, 1)=0, \quad \frac{\partial \theta(x^+, o)}{\partial r^+} = 0 \quad (7)
\]

with the dimensionless velocity profile being given by

\[
u^+ = \frac{2[1-r^{+2}-2c(1-r^{+})]}{1-\frac{4}{3}c+\frac{c^4}{3}} \quad (8)
\]

\[
= \frac{2(1-c)^2}{1-\frac{4}{3}c+\frac{c^4}{3}} \quad \text{for} \quad 0 \leq r^+ \leq c
\]

The classical separation of variables solution to Eq. (7) leads to

\[
\theta(x^+, r^+) = \sum_{n=0}^{\infty} C_n R_n(r^+) \exp(-\lambda_n^2 x^+) \quad (9)
\]

where \( C_n \) is a constant to be determined from the boundary conditions and \( R_n(r^+) \) and \( \lambda_n \) are eigenfunctions and eigenvalues respectively that are determined from the solution of

\[
\frac{d}{dr^+} \left( r^+ \frac{dR_n}{dr^+} \right) + \frac{\lambda_n^2}{2} u^+ r^+ R_n = 0 \quad R_n(1) = 0, \quad \frac{dR_n(0)}{dr^+} = 0 \quad (10)
\]

From the orthogonality condition,
A more convenient constant $G_n$ will be defined as

$$G_n = -\frac{C_n}{2} \frac{dR_n(1)}{dr^+} = \frac{[dR_n(1)/dr^+]^2/2}{\lambda_n^2} \frac{1}{2} \int_0^1 u^+ r^+ R_n^2 dr^+$$

From Eq. (9), several useful heat transfer parameters can be developed. The dimensionless bulk fluid temperature is

$$\theta_b(x^+) = \frac{t_b-t_b}{t_0-t_e} = 2 \int_0^1 u^+ \theta r^+ dr^+ = 8 \sum_{n=0}^\infty \frac{G_n}{\lambda_n^2} \exp(-\lambda_n^2 x^+)$$

where $t_b$ is the bulk fluid or mixing cup temperature. Defining the local heat transfer coefficient in terms of the local temperature difference $(t_b-t_0)$, the local Nusselt number becomes

$$Nu_x = \frac{h_x D}{k} = \frac{-2}{\theta_b} \frac{\partial \theta(x,1)}{\partial r^+} = 4 \sum_{n=0}^\infty \frac{G_n}{\lambda_n^2} \exp(-\lambda_n^2 x^+)$$

The average $Nu$ between the entrance and any arbitrary $x^+$ is given quite simply by

$$Nu_m(x^+) = \frac{1}{x^+} \int_0^{x^+} Nu_x dx^+ = \frac{1}{2x^+} \ln(1/\theta_b)$$

Eq. (10) is the classical Sturm-Liouville problem. A closed form analytical solution exists for plug flow ($c=1$, see Burmeister [5] for a discussion), Sellars, Tribus, and Klein [6] developed an approximate solution for laminar Newtonian
flow (c=0), and Wissler and Schechter [3] numerically
determined the first seven eigenvalues and eigenfunctions for
c=0.0, 0.25, 0.5, 0.75, and 1.0. Additional works are
referenced in [1]. The number of eigenvalues and
eigenfunctions reported by Wissler and Schechter [3] were found
to be inadequate for small values of x+ and the calculations
were extended to include the first 60 eigenvalues for c=0.0,
0.2, 0.4, 0.6, 0.8, 1.0.

The general Sturm-Liouville problem can be written as

\[ \frac{d}{dx} \left( p(x) \frac{d}{dx} \psi(x) \right) + (q(x) + \lambda r(x)) \psi(x) = 0 \quad a \leq x \leq b \]  \hspace{1cm} (16)

with boundary conditions of the form

\[ A_1 \psi(a) + A_2 p(a) \frac{d}{dx} \psi(a) = 0 \]
\[ B_1 \psi(b) + B_2 p(b) \frac{d}{dx} \psi(b) = 0 \]  \hspace{1cm} (17)

where p(x), q(x), and r(x) are arbitrary functions and \( \psi(x) \) is
the eigenfunction. The numerical results presented in this
article were produced by the SLEIGN code, described by Bailey
[7]. This code internally transforms the independent variable
x onto the interval (-1,1). Next, the second order
differential equation given by Eq. (16) is replaced (within the
code) by an equivalent system of two first order equations for
the new dependent variables \( p(x) \) and \( \psi(x) \) defined by

\[ \psi(x) = p(x) \sin \phi(x) \]
\[ p(x) \psi'(x) = z p(x) \cos \phi(x) \]  \hspace{1cm} (18)

where z is a scaling factor determined by the code. If z=1,
this is known as the Prufer transformation [7]. The eigenvalue
\( \lambda \) is then determined by numerically integrating the transformed
version of Eq. (16) from both boundaries toward the interior of
the internal (a,b) with an assumed \( \lambda \). The integration is ter-
minated at an interior point \( x=M \) and the solution from the left
\( \psi_L(M; \lambda) \) is compared with the solution from the right \( \psi_R(M; \lambda) \). During both the "left" and "right" integration process, the correct boundary conditions are always used. The code automatically chooses the match point \( x=M \), picks an initial guess for \( \lambda \) and adjusts \( \lambda \) until \( \psi_L = \psi_R \) within a user specified tolerance. The code has been extensively tested and additional details can be found in Bailey [7].

RESULTS

Table 1 presents numerical results for the local Nusselt number \( (\text{Nu}_X) \), average Nusselt number \( \text{Nu}_m \), and bulk fluid temperature as a function of the dimensionless entry length \( x^+ \). All calculations were performed on a CDC Cyber 170/Model 855 computer using single precision arithmetic (nominally 14 1/2 digits). The series for \( \text{Nu}_X \) converges more slowly than that for \( \theta_B \). A relative convergence criteria of \( 10^{-6} \) on the last term (normalized by the partial sum) was used. Sixty eigenvalues were adequate for convergence for all values of \( x^+ \) except 0.0001; for this \( x^+ \), the relative error was typically less than \( 7 \times 10^{-5} \) for all values of \( c \). The numerical results for \( c=1.0 \) were compared with the analytical solution; for this case, the eigenvalues are the roots of \( J_0(\lambda/n/\sqrt{2}) = 0 \) and the eigenfunctions are \( R_n(r^+) = J_0(\lambda/n/\sqrt{2} r^+) \).

The results from SLEIGN were identical to the analytical solution for the number of significant digits printed, except for \( x^+ = 0.0001 \). For example, the analytical result was \( \text{Nu}_X = 81.352 \) while the numerical result was 81.365. The \( c=1 \) (plug flow) results were also compared with those presented in Burmeister [5]; exact agreement was obtained for large \( x^+ \) but it appears that the results of [4] are not accurate at small \( x^+ \).

Sellars, Tribus, and Klein [6] developed an approximate solution for \( c=0 \) (laminar Newtonian flow) and their results for \( \text{Nu}_X, \text{Nu}_m, \) and \( \theta_B \) are tabulated in Kays and Crawford [8] and Burmeister [9]. Again, these results do not appear to be accurate at small \( x^+ \).

The results of Table 1 are also presented graphically in Figures 2-4. For \( c \) near zero, \( \text{Nu} \) and the bulk fluid
temperature are not very sensitive to $c$; for $c$ near unity, the computed results are much more sensitive to $c$. These results stem from the dependence of the velocity profile on $c$ (see Fig. 1).

CONCLUSIONS

The numerical solution of the Graetz problem of the development of the thermal boundary layer within a tube for laminar fully developed velocity profile under a constant wall temperature boundary condition was presented for a Bingham plastic. Local Nusselt number, average Nusselt number and bulk fluid temperature were presented as a function of dimensionless distance from the inlet. The results for plug flow agree with the analytical solution, and the laminar Newtonian flow ($c=0$) results of this work appear to be more accurate than those available in the literature.
Table 1  Heat Transfer Results for Developing Flow of Bingham Plastic in a Tube with Constant Wall Temperature

c=1.0 (plug flow)

<table>
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<th>Nu_x</th>
<th>Nu_m</th>
<th>θ_b</th>
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<td>81.352</td>
<td>161.146</td>
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<td>0.0002</td>
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\( c=0.8 \)

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\( c=0.6 \)

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Table 1 Heat Transfer Results for Developing Flow of Bingham Plastic in a Tube with Constant Wall Temperature (Cont)

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Table 1 Heat Transfer Results for Developing Flow of Bingham Plastic in a Tube with Constant Wall Temperature (Cont)

\( c=0.0 \) (Laminar Newtonian)

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REFERENCES


Figure 1  Dimensionless Velocity Profile for Fully Developed Flow of a Bingham Plastic in Circular Tube
\( \eta = \tau_y / \tau_w \)
Figure 2 Variation of Local Nusselt No ($\text{Nu}_x$) with Dimensionless Axial Distance $\left(\frac{x/r_0}{\text{Pe}}\right)$ for $c=0.0$, 0.2, 0.4, 0.6, 0.8, and 1.0.
Figure 3  Variation of Mean Nusselt No $(Nu_m)$ with Dimensionless Axial Distance $\frac{x/r_0}{Pe}$ for $c=0.0$, 0.2, 0.4, 0.6, 0.8, and 1.0.
Figure 4  Variation of Dimensionless Bulk Fluid Temperature ($\theta_b$) with Dimensionless Axial Distance $\frac{x/r_o}{Pe}$ for $c=0.0$, 0.2, 0.4, 0.6, 0.8, and 1.0.
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