Modelling and control of an electromechanical steering system in full vehicle models

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The manuscript was received on 10 September 2004 and was accepted after revision for publication on 12 December 2005.
DOI: 10.1243/09596518JSCE96

Abstract: In the automotive industry, electrical and electromechanical components and systems become more and more important. In comparison with commonly used mechanical and hydraulic systems they offer a large number of advantages with respect to efficiency and flexibility, for example. Therefore, conventional hydraulic steering systems are being replaced more and more with electromechanical ones. Currently, different concepts of electromechanical steering systems are being developed. In this work an electromechanical steering system with double pinions is modelled based on a uniform theory for discrete electromechanical systems. This steering system is implemented into a multibody full vehicle model and a control scheme has been developed. Subsequently, the performance of the whole electromechanical system, and especially the behaviour of the controller, has been tested with different handling manoeuvres.

Keywords: electromechanical steering system, electromechanical systems, multibody vehicle model, full vehicle model, driver model, vector control, field-oriented control

1 INTRODUCTION

Currently, an increasing importance is attached to electromechanical components in the automotive industry. An outstanding example is the electromechanical steering system which is characterized by efficiency, steering comfort, adaptability, and environmental friendliness. It already fulfils a large number of requirements for steering systems of the future [1]. In the case of electromechanical steering different concepts are known [2, 3]. In this paper, the concept with double pinions as shown in Fig. 1 is considered. This kind of steering system is equipped with two pinions. While the steering pinion transfers the steering torque applied by the driver, the torque generated by the servo motor is introduced over an additional servo pinion into the gear rack [4]. As a fundamental property of this construction, a mechanical connection always survives between steering wheel and gear rack. Thus, in case of a failure of the servo motor the vehicle can still be steered mechanically.

In this research work, an asynchronous electrical machine is used as the servo motor. Due to its brushless design the asynchronous machine offers many advantages and is a very compact and durable functional unit. For the operation of this steering system different input signals are required. The most important signal is from the torque sensor on the torsion bar, which corresponds with the hand torque of the driver applied on the steering wheel. A further important signal is the angular velocity of the rotor, which is necessary for a precise control of the asynchronous machine. This signal is measured by means of an angular velocity sensor. An optimally adapted steering support can be achieved by taking further information into account, e.g. the vehicle velocity or the steering angle. In order to generate various steering functions, the input signals are processed by an algorithm in the control unit. Subjects of the present research work are the modelling and
control of such an electromechanical steering system in a multibody full vehicle model. The behaviour of the whole system is tested using different handling manoeuvres with the help of a driver model.

2 MODELLING OF THE ELECTROMECHANICAL STEERING SYSTEM

The steering system as shown above is a typical electromechanical system (EMS). It represents a physical heterogeneous structure, which can be characterized by the interactions between electromagnetic fields and mechanical objects [5]. The dynamical behaviour and interactions within such a system can be described by coupling the methods of multibody dynamics with the Kirchhoff theory [6, 7]. For this purpose the approach of generating uniform Lagrangean motion equations for the whole EMS works well [7].

2.1 Lagrangean formalism for discrete EMS

To describe the dynamical behaviour of discrete electromechanical systems with the Lagrangean formalism, its application has been transferred first to pure electrical systems with concentrated parameters, such as resistors, capacitors, and inductors [8]. The topology of the electrical system can be represented by a so-called network graph $\Gamma$. The set \{\bar{q}_j, x^s_l\} \in \Gamma; s = 1, \ldots, 6, k = 1, \ldots, K\} is called a position of the EMS, where $\bar{q}_j$ are the branch charges and $x^s_l$ the mechanical coordinates of a given number of $K$ rigid bodies. The kinematics of the EMS is determined by the constraint conditions, which are given by Kirchhoff’s current law of the electrical network and the geometric connections between the rigid bodies. The vectors of the branch charges $\vec{q} = (\bar{q}_j)$ and the mechanical coordinates $\vec{x} = (x^s_l)$ which satisfy the constraint conditions of the EMS at the time $t$ can be written as

\begin{align*}
\vec{q} &= Aq^0 + \vec{q}^0(t) \\
\vec{x} &= \vec{x}(q^0, t)
\end{align*}

where $q^0$ is the vector of electrical generalized coordinates (charges of the fundamental loops), $A$ is the fundamental loop matrix describing the linear dependence between the branch charges and the electrical generalized coordinates, and $q^0(t)$ represents the charges of the current sources in the electrical network. Furthermore, $q^0$ is the vector of mechanical generalized coordinates. The number of components of the generalized electrical and mechanical coordinate vectors $q^0$ and $q^0$ is called quasi degree of freedom $n$ of the discrete EMS. Then, the vector $q = (q^0; q^0)$ can be interpreted as the representing point of the discrete EMS in the $n$-dimensional configuration space $\mathbb{R}^n$. The motion equations of a discrete EMS can be written in the form

\begin{align*}
M(t, q^0)\ddot{q}^0 + K(t, q^0, \dot{q}^0) + \dot{Q}(t, q^0, \dot{q}^0) &= 0 \\
L(t, q^0)\ddot{q}^0 + G(t, q^0, \dot{q}^0, \dot{q}^0) + R(t, q^0)\dot{q}^0 &= 0 \\
C(t, q^0)\dot{q}^0 + V(t, q^0) &= 0
\end{align*}

where $M(t, q^0)$ is the generalized mass matrix, $K(t, q^0, \dot{q}^0)$ is the vector of generalized gyroscopic/Coriolis forces, $Q(t, q^0, \dot{q}^0)$ is the vector of the generalized applied forces having a pure mechanical origin (e.g. gravity, springs, frictions, and dampers), and $G(t, q^0, \dot{q}^0, \dot{q}^0)$ is the vector of applied forces caused by electromagnetic fields. The matrix $L(t, q^0)$ contains the generalized inductances and $G(t, q^0, \dot{q}^0, \dot{q}^0)$ is the vector of generalized gyroscopic/Coriolis voltages. $R(t, q^0)$ is the matrix of generalized resistances, $C(t, q^0)$ represents the generalized capacitances, and $V(t, q^0)$ denotes the vector of the generalized applied voltages.

Obviously, the mechanical part (1) of the motion equations contains additional forces depending on the generalized electrical coordinates. On the other hand, the coefficient matrices appearing in the electrical part (2) of the motion equations can depend on the mechanical generalized coordinates. Thus, the coupling between the mechanical and electrical parts of the system is completely represented by this theory. The mathematical and physical foundations and details of this uniform approach can be found in reference [9].

The theory of the Lagrangean formalism for discrete electromechanical systems is implemented in the simulation program alaska, which was developed at the Institute of Mechatronics in Chemnitz, Germany. The Lagrangean motion equations (1) and (2) of an EMS can be generated automatically in alaska.
2.2 Modelling of the induction machine

In general an induction machine consists of a stator and a rotor, which are connected by a revolute joint. An induction machine possessing a three-phase winding system in the stator and a further three-phase winding system in the rotor is of special interest in this research work. In order to formulate the model for the induction machine the following conditions are assumed to be fulfilled:

1. The windings can be replaced by concentrated windings.
2. The magnetization characteristic is linear and without saturation.
3. The magnetomechanical interactions are determined by the field distribution in the air gap between the rotor and stator.
4. Dissipative core losses and temperature dependence of the resistances and inductances are neglected.

Figure 2 shows the model of an idealized three-phase induction machine. In the following considerations, the indices s and r denote quantities referring to stator and rotor, while a, b, and c denote the three phases. The inductances arranged in the stator and rotor windings are then represented by three coils staggered by \(2\pi/3\) in each case. Each of these six coils forms a fundamental loop together with an Ohm's resistance and a voltage source. If no voltage source is applied in the rotor windings, a so-called asynchronous machine is obtained.

With the help of the electromechanical modelling components contained in alaska, e.g. capacitors (C), coils (L), resistors (R), and voltage sources (V), an electrical machine of this kind can be modelled easily. The network graph of a three-phase induction machine is shown in Fig. 3.

From the above network graph the fundamental loop matrix can be written as

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}^T
\]

This three-phase induction machine has seven quasi degrees of freedom. Accordingly,

\[
\begin{align*}
q &= (q^f; \dot{q}^f) = (q_1, q_2, q_3, q_4, q_5, q_6, q_7) \\
&= (q_{as}, q_{bs}, q_{cs}, q_{ar}, q_{br}, q_{cr}, \theta_{rm})
\end{align*}
\]

is the vector of the generalized coordinates, where \(q_j, j \in \{1, \ldots, 6\}\), denotes the charge in the fundamental loop \(j\) and \(q_7\) is the mechanical rotation angle \(\theta_{rm}\) of the rotor. Consequently,

\[
\begin{align*}
\dot{q} &= (\dot{q}^f; \dot{q}^f) = (\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4, \dot{q}_5, \dot{q}_6, \dot{q}_7) \\
&= (\dot{i}_{as}, \dot{i}_{bs}, \dot{i}_{cs}, \dot{i}_{ar}, \dot{i}_{br}, \dot{i}_{cr}, \omega_{rm})
\end{align*}
\]

is the vector of the generalized velocities, where \(\dot{q}_j, j \in \{1, \ldots, 6\}\), denotes the current in the fundamental loop \(j\) and \(\dot{q}_7\) is the mechanical angular velocity \(\omega_{rm}\) of the rotor.

If the induction machine possesses \(p\) poles (\(p_p = p/2\) pole pairs), a so-called electrical angle \(\theta_i\) of the rotor can be introduced in accordance with

\[
\theta_i = p_p \theta_{rm}
\]

Consequently, the electrical angular velocity is defined by

\[
\omega_i = p_p \omega_{rm}
\]

The parameters describing the physical properties of the electrical components are still required for the complete description of the induction machine. In

\[
\Gamma = \{1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12\}
\]
accordance with the fundamental assumptions, the
Ohm's resistances \( r_s \) of the stator windings and \( r_r \) of
the rotor windings are constant. The inductances of
the induction machine can be written in matrix form as
\[
L = \begin{bmatrix}
L_{0s} + L_{ss} & L_{sm} & L_{st} & L_{0t} & L_{pt} & L_{mt}
\end{bmatrix}
\]

The parameter \( L_{ss} \) denotes the self-inductance of the
stator windings and \( L_{sm} = L_{ss} \cos(2\pi/3) \) the mutual
inductance between two stator windings. \( L_{st} \) is the
self-inductance of the rotor windings and \( L_{ts} = L_{st} \cos(2\pi/3) \) the mutual
inductance between two rotor windings. \( L_{0s} \) is the
leakage inductance of the stator windings and \( L_{0t} \) the leakage inductance of
the rotor windings. The relations \( L_{0s} = L_{ss} \cos\theta_t, \)
\( L_{pt} = L_{ss} \cos(\theta_t + 2\pi/3), \)
\( L_{mt} = L_{ss} \cos(\theta_t - 2\pi/3) \) are con-
considered, where \( L_{ss} \) is the peak value of the mutual
inductance between the stator and rotor windings.

To generate the differential equations of an EMS
the program alaska requires the derivatives of the
parameters describing the physical properties of the
electrical components with respect to both the
mechanical coordinates and the time. It is recognized
that the parameters \( L_{0s}, L_{pt}, \) and \( L_{mt} \) of the inductance
matrix depend on the time-dependent rotor angle \( \theta_t \).
Therefore, the partial derivative of the inductance
matrix with respect to the rotor angle \( \theta_t \) is not zero
and must be indicated as
\[
\frac{dL}{d\theta_t} = \begin{bmatrix}
0 & 0 & 0 & L_{0t} & L_{pt} & L_{mt}
0 & 0 & 0 & L_{0t} & L_{pt} & L_{mt}
0 & 0 & 0 & L_{0t} & L_{pt} & L_{mt}
L_{0s} & L_{pt} & L_{0t} & 0 & 0 & 0
L_{pt} & L_{0st} & L_{0pt} & 0 & 0 & 0
L_{mt} & L_{pt} & 0 & 0 & 0 & 0
\end{bmatrix}
\]

with \( L_{0s} = -p_y L_{ts} \sin\theta_t, \)
\( L_{pt} = -p_y L_{ts} \sin(\theta_t + 2\pi/3) \)
and \( L_{mt} = -p_y L_{ts} \sin(\theta_t - 2\pi/3). \) The time derivative of the inductance matrix can then be written as
\[
\frac{dL}{dt} = \omega_{tm} = \begin{bmatrix}
0 & 0 & 0 & L_{0t} & L_{pt} & L_{mt}
0 & 0 & 0 & L_{0t} & L_{pt} & L_{mt}
0 & 0 & 0 & L_{0t} & L_{pt} & L_{mt}
L_{0s} & L_{pt} & L_{0t} & 0 & 0 & 0
L_{pt} & L_{0st} & L_{0pt} & 0 & 0 & 0
L_{mt} & L_{pt} & 0 & 0 & 0 & 0
\end{bmatrix}
\]

2.3 Electromechanical steering system in a full vehicle model

The electrical machine as discussed above was
implemented in the steering system of a multibody
full vehicle model according to the double pinion
concept. For the purpose of describing the forces
between the road and the wheel hub, the tyre model
RMOD-K is used (see references [10] to [12]). The
parking torque and the steering torque at a standstill
of the vehicle can also be calculated with this tyre
model. The multidisc full vehicle model in combination
with the electromechanical steering system is shown in Fig. 4.

The differential equations of this model are
generated automatically by the program alaska. Due to
the inductances the mechanical coordinate \( \theta_t \) enters the electrical part (2) of the motion equations. On the other hand, the electromagnetic torque
occurs as an additional force in the mechanical
part (1) of the motion equations. The simultaneous
solution of the motion equations of both model
parts guarantees the correct description of all
electromechanical interactions occurring in the
model.

2.4 A driver model for testing the electromechanical steering system

In order to simulate the dynamical behaviour of the
electromechanical steering system with different,
more or less complicated driving manoeuvres, a
driver model is used. This driver model consists of
both a trace controller and a velocity controller. It
has proven itself in various applications [12]. The concept of the trace controller, which works with
preview, is sketched in Fig. 5. The presented target trace \( C_t(s) \) that the vehicle
data must to follow lies in the \( (E_x, E_y) \) plane of the
 inertial system and uses the arc length \( s \) as the
representation or curve parameter. The local

Fig. 4 A multidisc full vehicle model with electromechanical steering system
 Electric machine is denoted as target trace. The Charlesworth Group, Wakefield 01924 204830

The steering wheel angle \( \theta \) required for the trace control is determined by means of a first-order differential equation linearly dependent on the lateral deviation \( \delta \) and the direction deviation \( \psi \):

\[
\phi(t) = \frac{1}{\tau_e} [k_v \psi(s_v, t) + k_d \delta(t) - \phi(t)]
\]

The behaviour of the trace controller can be adjusted by the time constant \( \tau_e \) and the two gain factors \( k_v \) and \( k_d \).

The velocity controller is realized by regulating the drive torque \( M_L \) depending on the difference between the current velocity of the vehicle in forward direction \( v_s = \vec{v} \cdot \vec{e}_f \) and the desired velocity \( v_{s,\text{des}} \). For this purpose a simple proportional controller is used

\[
M_L = k_A (v_{s,\text{des}} - v_s),
\]

where \( k_A \) is a gain factor of the drive torque regulation. A detailed description of this driver model can be found in reference [13].

2.5 Determination of the entire steering torque and the desired support torque

A main characteristic of the electromechanical steering system is that the steering motion is given at each time by the driver. The corresponding joint reaction forces enforcing the given motion can be calculated by means of inverse dynamics of the mechanical subsystem. To this end, the generalized mechanical coordinates \( q^l \) are divided into so-called intrinsic coordinates \( q^1 \), whose motion is prescribed, and external coordinates \( q^2 \). Then, the motion equations describing the dynamics of the mechanical subsystem can be formulated as

\[
M_{q^1} \ddot{q}^1 + M_{q^2} \ddot{q}^2 + \kappa^1 + Q^1 + Q^2 = 0
\]

(7)

\[
M_{q^1} \ddot{q}^1 + M_{q^2} \ddot{q}^2 + \kappa^2 + Q^1 + Q^2 = 0
\]

(8)

If the rheonomic constraints of the intrinsic coordinates are given as

\[
\bar{q}^1 = \bar{q}^1(t)
\]

the reaction forces \( Q^R \) can be determined from equations (7) and (8) using inverse dynamics (see also reference [14]). The calculated reaction force enforces the prescribed motion of the steering system and corresponds to the entire necessary steering torque \( M_L \). The task of the electrical machine of the electromechanical steering system is to supply a certain part of the entire steering torque. In the following considerations the support torque generated by the electrical machine is denoted as \( M_E \). The remaining steering torque \( M_L - M_E \) corresponds with the torque stamped at the torsion bar and will therefore be called the hand torque \( M_H \). The target relationship between the hand torque and the support torque is determined or estimated by the development engineer in accordance with the desired steering feeling. It is given, for example, in the form of a characteristic field as shown in Fig. 6. The characteristic field matches the desired value of the support torque \( M_E \) to that of the vehicle velocity and the hand torque.

If \( i \) denotes the gear ratio between the rotor of the electrical machine and the steering wheel, the electrical machine must generate a torque \( m_E = i M_E \).
3 CONTROL OF THE ELECTROMECHANICAL STEERING SYSTEM

Since in this steering system the rotor of the electrical machine is connected mechanically with the steering wheel, the rotation of the rotor is determined by the steering wheel motion initiated by the driver and cannot be controlled. Therefore, the control task of the electromechanical steering system is to produce the desired torque as accurately as possible at any given angular velocity of the rotor. For this highly dynamical servo drive it is necessary to employ the dynamic model of the electrical machine during the design of the controller. The presuppositions, to supply the electrical machine with voltages of controllable amplitude, frequency, and phase, can be easily fulfilled nowadays by efficient power switches and fast microprocessors.

3.1 Model equations of an induction machine in the a–b–c reference frame

According to the electrical part (2) of the motion equations the dynamic model of the induction machine can be expressed explicitly in the so-called a–b–c reference frame as follows.

Voltage equations for the stator

$$
\begin{align*}
    u_{as} &= \frac{d\psi_{as}}{dt} + i_{as} r_s \\
    u_{bs} &= \frac{d\psi_{bs}}{dt} + i_{bs} r_s \\
    u_{cs} &= \frac{d\psi_{cs}}{dt} + i_{cs} r_s
\end{align*}
$$

(9)

Voltage equations for the rotor

$$
\begin{align*}
    u_{ar} &= \frac{d\psi_{ar}}{dt} + i_{ar} r_t \\
    u_{br} &= \frac{d\psi_{br}}{dt} + i_{br} r_t \\
    u_{cr} &= \frac{d\psi_{cr}}{dt} + i_{cr} r_t
\end{align*}
$$

(10)

Constitutive equations (flux–current relationships)

$$
\begin{align*}
    \begin{bmatrix}
        \psi_{as} \\
        \psi_{bs} \\
        \psi_{cs} \\
        \psi_{ar} \\
        \psi_{br} \\
        \psi_{cr}
    \end{bmatrix}
    &=
    \begin{bmatrix}
        L_{as} & L_{as} & L_{as} & L_{as} & L_{as} & L_{as} \\
        L_{as} & L_{as} + L_{as} & L_{as} & L_{as} & L_{as} & L_{as} \\
        L_{as} & L_{as} & L_{as} & L_{as} & L_{as} & L_{as} \\
        L_{as} & L_{as} & L_{as} & L_{as} & L_{as} & L_{as} \\
        L_{as} & L_{as} & L_{as} & L_{as} & L_{as} & L_{as} \\
        L_{as} & L_{as} & L_{as} & L_{as} & L_{as} & L_{as}
    \end{bmatrix}
    \begin{bmatrix}
        i_{as} \\
        i_{bs} \\
        i_{cs} \\
        i_{ar} \\
        i_{br} \\
        i_{cr}
    \end{bmatrix}
\end{align*}
$$

(11)

In the equations (9) and (10), $u_{ik}$ with $i \in \{a, b, c\}$ and $k \in \{s, r\}$ denote the externally supplied voltages. It should be noted that $u_{0r} = 0$ holds for asynchronous electrical machines. The quantities $\psi_{ik}$ denote the flux linkages. The electromagnetically generated air gap torque, which corresponds to the electromechanical steering system is to produce the desired torque as accurately as possible at any given angular velocity of the rotor. For this highly dynamical servo drive it is necessary to employ the dynamic model of the electrical machine during the design of the controller. The presuppositions, to supply the electrical machine with voltages of controllable amplitude, frequency, and phase, can be easily fulfilled nowadays by efficient power switches and fast microprocessors.

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$$
    m_E = \frac{1}{2} \sum_{i=r} \sum_{k=s} \frac{\partial \psi_{ik}^{\prime}}{\partial \theta_{rm}} i_{ik}
$$

Using the matrix $\frac{\partial L}{\partial \theta_{rm}}$, this expression simplifies to (see also reference [15])

$$
\begin{align*}
    m_E &= \frac{p_e}{\sqrt{3}} (\psi_{as} i_{bs} - \psi_{bs} i_{as}) + \frac{p_e}{\sqrt{3}} (\psi_{ar} i_{cs} - \psi_{cs} i_{ar}) \\
    &+ \frac{p_e}{\sqrt{3}} (\psi_{cr} i_{as} - \psi_{as} i_{cr})
\end{align*}
$$

(12)
This system of coupled differential equations (9) to (12) shows that the induction machine is a non-linear, multivariable system. In order to control it, a field-oriented scheme [16, 17] is employed in this research work. The decoupling of the equations can be achieved by substituting the \(a-b-c\) reference frame with a so-called \(q-d-0\) reference frame.

### 3.2 Model equations of an induction machine in the \(q-d-0\) reference frame

A so-called \(q-d-0\) reference frame rotating with an electrical angular velocity \(\omega\) in the same direction as the rotor is shown in Fig. 7. In the special case of \(\omega = 0\) there is a stationary reference frame. Setting \(\omega = \omega_s\), where \(\omega_s\) is the so-called synchronous electrical angular velocity of the magnetic field of the stator, a synchronously rotating reference frame can be obtained. The transformation of a quantity from the \(a-b-c\) to the \(q-d-0\) reference frame can be carried out with the following matrix

\[
T(\varphi) = \frac{2}{3} \begin{bmatrix}
\cos \varphi & \cos(\varphi - 2\pi/3) & \cos(\varphi + 2\pi/3) \\
\sin \varphi & \sin(\varphi - 2\pi/3) & \sin(\varphi + 2\pi/3) \\
1/2 & 1/2 & 1/2
\end{bmatrix}
\]

The matrix \(T(\varphi)\) in fact describes a linear transformation in the velocity space of the planar electrical machine. Looking at Fig. 7, it is clear that \(\varphi = \theta = \int_0^t \omega(t) \, dt + \theta(0)\) applies to the quantities belonging to the stator and \(\varphi = \theta - \theta_r\), to the quantities of the rotor. Applying the matrix \(T(\varphi)\) to the stator or rotor quantities, the following transformation relation holds

\[
\begin{bmatrix}
f_q \\ f_d \\ f_0
\end{bmatrix} = T(\varphi) \begin{bmatrix}
f_q \\ f_d \\ f_0
\end{bmatrix}
\]

The quantities to be transformed may be voltages, currents, or flux linkages. The inverse transformation matrix from the \(q-d-0\) to the \(a-b-c\) reference frame can be written as

\[
T(\varphi)^{-1} = \begin{bmatrix}
\cos \varphi & \sin \varphi & 1 \\
\cos(\varphi - 2\pi/3) & \sin(\varphi - 2\pi/3) & 1 \\
\cos(\varphi + 2\pi/3) & \sin(\varphi + 2\pi/3) & 1
\end{bmatrix}
\]

Applying this transformation to the voltage and constitutive equations in the \(a-b-c\) reference frame, the corresponding equations in the \(q-d-0\) reference frame are the following ones.

**Voltage equations for the stator**

\[
\begin{align*}
u_{qs} &= \frac{d\Psi_{qs}}{dt} + \omega \Psi_{ds} + r_i i_{qs} \\
u_{ds} &= \frac{d\Psi_{ds}}{dt} - \omega \Psi_{qs} + r_i i_{ds} \\
u_{os} &= \frac{d\Psi_{os}}{dt} + r_i i_{os}
\end{align*}
\]

**Voltage equations for the rotor**

\[
\begin{align*}
u_{qr} &= \frac{d\Psi_{qr}}{dt} + (\omega - \omega_r) \Psi_{dr} + r_i i_{qr} \\
u_{dr} &= \frac{d\Psi_{dr}}{dt} - (\omega - \omega_r) \Psi_{qr} + r_i i_{dr} \\
u_{or} &= \frac{d\Psi_{or}}{dt} + r_i i_{or}
\end{align*}
\]

**Constitutive equations**

\[
\begin{bmatrix}
\Psi_{qs} \\ \Psi_{ds} \\ \Psi_{os} \\ \Psi_{qr} \\ \Psi_{dr} \\ \Psi_{or}
\end{bmatrix} = \begin{bmatrix}
L_s & 0 & 0 & L_m & 0 & 0 \\
0 & L_s & 0 & 0 & L_m & 0 \\
0 & 0 & L_s & 0 & 0 & 0 \\
L_m & 0 & 0 & L'_r & 0 & 0 \\
0 & L_m & 0 & 0 & L_r & 0 \\
0 & 0 & 0 & 0 & L_r & 0
\end{bmatrix} \begin{bmatrix}
i_{qs} \\ i_{ds} \\ i_{os} \\ i_{qr} \\ i_{dr} \\ i_{or}
\end{bmatrix}
\]

In the equations (13), (14), and (15) the definitions \(L'_r = L_{rs} + 3L_{rs}/2\), \(L_r = L_{rs} + 3L_{rs}/2\), and \(L_m = 3L_{rs}/2\) and are taken into account. In terms of \(q-d-0\) quantities,
the air gap torque generated by the electrical machine can be expressed in the following form (see, for example, reference [18])

\[
m_{E} = \frac{3}{2} p_{e} (\Psi_{q}^{e} i_{dt} - \Psi_{dt} i_{qe}) \tag{16}
\]

### 3.3 Field-oriented control of the asynchronous machine

Selecting, for example, a synchronously rotating \(q-d-0\) reference frame whose \(d\) axis is aligned with the rotor field (see Fig. 8, where all quantities in this reference frame are characterized by the index \(e\)), the \(q\) component of the rotor flux is always zero

\[
\Psi_{q}^{e} = L_{m} v_{q}^{e} + L_{r} i_{qe}^{e} = 0
\]

Hence

\[
i_{qe}^{e} = - \frac{L_{m} v_{q}^{e}}{L_{r}} \tag{17}
\]

With \(\Psi_{q}^{e} = 0\) the expression (16) for the air gap torque reduces to

\[
m_{E} = - \frac{3}{2} p_{e} \Psi_{dt} i_{qe} \tag{18}
\]

Substituting the current \(i_{qe}^{e}\) using equation (17), equation (18) can be written in the desired form

\[
m_{E} = \frac{3}{2} p_{e} L_{m} v_{q}^{e} i_{qe}^{e} \tag{19}
\]

which shows that the air gap torque can be controlled by adjusting either the \(q\) component of the stator current or the rotor flux linkage.

Since for asynchronous machines there are no voltages applied to the rotor windings, then \(u_{dt}^{e} = 0\). Taking \(\Psi_{q}^{e} = 0\) into account the voltage equation for the \(d\) axis of the rotor reduces to

\[
u_{dt}^{e} = \frac{d \Psi_{dt}^{e}}{dt} + r_{r} i_{qe}^{e} = 0 \tag{20}
\]

Inserting the flux linkage equation

\[
i_{dt}^{e} = \frac{\Psi_{dt}^{e} - L_{m} i_{qe}^{e}}{L_{r}}
\]

![Fig. 8 q-d-0 reference frame by rotor field orientation](image)

Equation (20) shows that the air gap torque can be calculated according to equation (16) as follows

\[
\Psi_{dt}^{e} = \frac{L_{r} i_{dt} \frac{d \Psi_{dt}^{e}}{dt} + \frac{1}{L_{m}} \Psi_{ds}^{e}}{r_{r} L_{m}} \tag{21}
\]

This relation means that the rotor flux linkage \(\Psi_{ds}^{e}\) can be controlled, in principle, by adjusting the \(d\) component of the stator current \(i_{ds}^{e}\). Since its dynamic behaviour is limited by the rotor circuit time constant \(L_{r} / r_{r}\), the control of \(\Psi_{ds}^{e}\) is not suitable for fast changes in torque. Considering equation (19), this task can be carried out more successfully by controlling the \(q\) component of the stator current \(i_{qs}^{e}\); if the rotor flux linkage \(\Psi_{qs}^{e}\) is not disturbed, the air gap torque can be independently adjusted by \(i_{qs}^{e}\) without delay.

If \(\Psi_{q}^{e} \equiv 0\), the voltage equation for the \(q\) axis of the rotor winding without any applied voltages (\(u_{dt}^{e} = 0\) reduces to

\[
u_{q}^{e} = (\omega_{e} - \omega_{r}) \Psi_{dt}^{e} + r_{r} i_{qe}^{e} = 0 \tag{22}
\]

Inserting equation (17) into equation (22) yields the following control condition

\[
\omega_{e} = \omega_{r} + \frac{r_{r} L_{m}}{L_{r}} \frac{\Psi_{ds}^{e}}{\Psi_{dt}^{e}} = \omega_{r} + \omega_{2} \tag{23}
\]

Equation (23) determines the angular velocity of the \(q-d-0\) reference frame, so that alignment of the \(d\) axis with the rotor field takes place and therefore \(\Psi_{q}^{e} \equiv 0\) is dynamically guaranteed.

The discussed field-oriented control scheme is an effective approach for decoupling the non-linear multivariable system of equations of an asynchronous machine. The aim of the control strategy is to keep the rotor flux \(\Psi_{ds}^{e}\) as exactly as possible through adjusting the \(d\) component of the stator current \(i_{ds}^{e}\) and to control the air gap torque by adjusting the \(q\) component of the stator current \(i_{qs}^{e}\), as well as the angular velocity \(\omega_{e}\). If a desired rotor flux \(\Psi_{ds}^{e}\) is given, the required \(d\) component of the stator current \(i_{ds}^{e}\) can be determined in accordance with equation (21) by

\[
i_{ds}^{e} = \frac{L_{r} i_{dt} \frac{d \Psi_{dt}^{e}}{dt} + \frac{1}{L_{m}} \Psi_{ds}^{e}}{r_{r} L_{m}} \tag{24}
\]

To generate the desired air gap torque \(m_{E}^{e}\) the required \(q\) component of the stator current \(i_{qs}^{e}\) can be calculated according to equation (19) as follows

\[
i_{qs}^{e} = \frac{2}{3} \frac{L_{r} i_{dt} \frac{d \Psi_{dt}^{e}}{dt} + \frac{1}{L_{m}} \Psi_{ds}^{e}}{r_{r} L_{m}} \tag{25}
\]
From equation (23) the following expression yields

\[ \omega_0^* = \omega_i + \frac{r_1 L_m}{L_i} \frac{\psi_m^*}{\psi_m^*} = \omega_i + \omega_L^* \tag{26} \]

The rotation angle \( \theta_0^* \) which determines the field orientation will be obtained by numerical integration of the angular velocity \( \omega_0^* \). This angle is used to transform the quantities from the \( q-d-0 \) to the \( a-b-c \) reference frame.

In this research work a controlled stator current supply is employed. The stator current components \( i_a^*, i_b^*, \) and \( i_c^* \) are realized by voltage supply with additional current regulations.

The block diagram of the control concept for the electromechanical steering system is shown in Fig. 9. In detail, an ISO lane change and a sinusoidal steering manoeuvre at standstill have been simulated. The desired value of the rotor flux is matched to the mechanical angular velocity of the rotor and is weakened beyond the base speed. Due to the physical limitations of power switches this field-weakening system is necessary in practical applications. In such a field-oriented control scheme the response speed of the electrical machine is limited in principle only by the delay of the current regulation.

4 RESULTS

In order to test and demonstrate the dynamical behaviour of the developed model and control scheme for the electromechanical steering system, a set of full vehicle simulations is carried out. Some numerical results, obtained by the simulation program alaska, are shown in Fig. 10. In detail, an ISO lane change and a sinusoidal steering manoeuvre at standstill have been simulated.

The results demonstrate that the desired support torque can be generated by the electrical machine in a sufficiently exact manner. A complete and correct

![Fig. 9 Control block diagram of the electromechanical steering system](image1)

![Fig. 10 Simulation results of the electromechanical steering system](image2)
description of all interactions between the mechanical and electrical subsystems of the steering system has been achieved through a uniform modelling technique of the whole system.

5 SUMMARY

In this research work, an electromechanical steering system is modelled on the basis of a uniform theory for discrete electromechanical systems. To control the electrical machine in this steering system, a field-oriented control scheme is applied. For the purpose of validating and testing the electromechanical steering system, it is implemented in a multibody full vehicle model. To this end, an ISO lane change and a standstill steering manoeuvre have been simulated. The numerical results show that the electrical machine generates the desired support torque with excellent accuracy. Based on this conceptual work the investigation of other interesting steering concepts is possible (e.g. a variable steering gear ratio, vehicle dynamics control with active steering, etc.). Such an electromechanical steering system may also serve as a basis for the implementation of steer by wire functionalities.

REFERENCES

1 Foth, J., Gazyakan, Ü., Dominke, P., and Ruck, G. Steering systems for future requirements. In European Automotive Congress (EAECC), Barcelona, Spain, 1999.

APPENDIX

Notation

A  fundamental loop matrix
C  matrix of the generalized capacitances
e_n  local normal vector of the target trace
e_t  local tangent vector of the target trace
e_v  preview vector of the trace controller
e_s  longitudinal axis of the vehicle
G  vector of the generalized gyroscopic/Coriolis voltages
i  gear ratio
i_{lk}  current in a fundamental loop
k  gain factor
Electromechanical steering system in full vehicle models

\[ K \] vector of the generalized gyroscopic/Coriolis forces

\[ L \] matrix of the generalized inductances

\[ L_{nt} \] leakage inductance of the rotor winding

\[ L_{ns} \] leakage inductance of the stator winding

\[ L_{rm} \] mutual inductance between two rotor windings

\[ L_{rr} \] self-inductance of the rotor winding

\[ L_{sm} \] mutual inductance between two stator windings

\[ L_{ns} \] peak value of the mutual inductance between stator and rotor windings

\[ m_g \] self-inductance of the stator winding

\[ m_t \] air gap torque

\[ M \] generalized mass matrix

\[ M_A \] drive torque of the power train

\[ M_E \] support torque

\[ M_H \] hand torque

\[ M_L \] entire steering torque

\[ p \] number of poles

\[ p_p \] number of pole pairs

\[ \dot{q} \] vector of the branch charges

\[ q^r \] vector of the mechanical generalized coordinates

\[ q^1 \] vector of the intrinsic coordinates

\[ q^2 \] vector of the external coordinates

\[ q^v \] vector of the electrical generalized coordinates

\[ q^p \] vector of the charges of current sources

\[ Q^r \] vector of the generalized applied forces

\[ Q^r' \] vector of the generalized applied forces (mechanical)

\[ Q^e' \] vector of the generalized applied forces (electrically caused)

\[ r_e \] Ohm's resistance of the rotor winding

\[ r_s \] Ohm's resistance of the stator winding

\[ R \] matrix of the generalized resistances

\[ s \] arc length

\[ s_v \] preview distance of the trace controller

\[ u_{lk} \] supply voltage in a fundamental loop

\[ v \] velocity vector of the vehicle-fixed reference point

\[ V \] vector of the generalized applied voltages

\[ \dot{x} \] vector of the mechanical coordinates

\[ \delta \] lateral deviation of the trace controller

\[ \theta_e \] electrical angle of the stator field

\[ \theta_i \] electrical angle of the rotor

\[ \theta_{rm} \] mechanical angle of the rotor

\[ \rho \] local curvature radius of the target trace

\[ \tau_v \] time constant of the trace controller

\[ \phi_s \] steering wheel angle

\[ \psi \] direction deviation of the trace controller

\[ \psi^r_{lk} \] flux linkage in a fundamental loop

\[ \omega \] electrical angular velocity of the q-d-0 reference frame

\[ \omega_e \] electrical angular velocity of the stator field

\[ \omega_i \] electrical angular velocity of the rotor

\[ \omega_{rm} \] mechanical angular velocity of the rotor

\[ \omega_2 \] slip angular velocity of the rotor