Chapter 8

DECAY HEAT GENERATION IN FISSION REACTORS

3/22/2011

1. INTRODUCTION

After a reactor core is shut down, through the insertion of its control rods, heat continues to be generated by the decay of the fission products, even though the fission power would stop to be generated.

The fission products heat generation, also called afterheat or decay heat, would have to be extracted from the system; otherwise it would lead to fuel damage, steam-cladding interaction leading to hydrogen generation, melting or even vaporization of the core. It depends on the design of the nuclear power plant as shown in Fig. 1, particularly on its power density.

Light Water Reactors, LWRs as Pressusized Water Reactors, PWRS and Boiling Water Reactors, BWRs offer the same type of response.

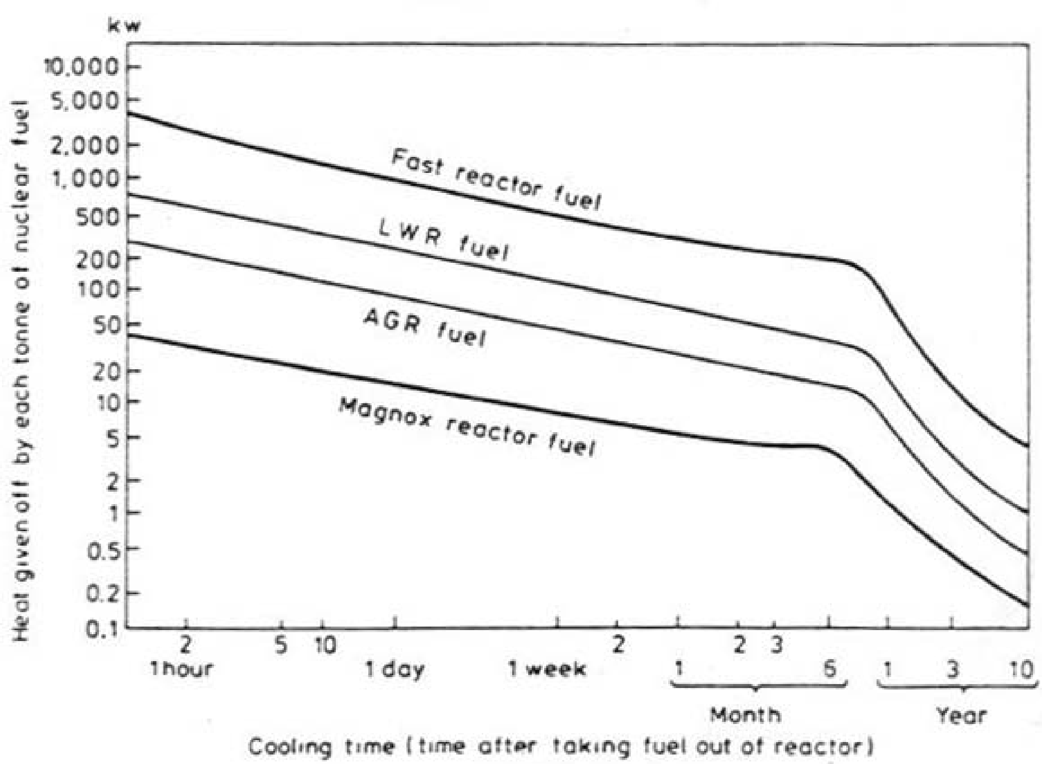

Figure 1. Power in thermal kilowatts per unit mass of reactor fuel in metric tonnes for different reactor designs. LWR: Light Water Reactor, AGR: Advanced Gas-cooled Reactor, Magnox: Magnesium alloy cladding reactor.

The decay heat power generation decreases rapidly as a function of time, but cooling provisions for dissipating it immediately after shutdown must be incorporated into the reactor's design. The power release from such a process must be estimated and accounted for in the safety design of nuclear power plants.

2. SOURCE TERMS

Two source terms contribute to the decay heat rates. One results from the rate of negative beta particles emission by the fission products given by:

$$R_{\beta}(T) = 3.8\times 10^{-6} T^{-1.2}\left[\frac{\text{particles}}{\text{sec.fission}}\right] \tag{1}$$

The second results from the rate of gamma photons emissions by the fission products, given by:

$$R_{\gamma}(T) = 1.9\times 10^{-6} T^{-1.2}\left[\frac{\text{photons}}{\text{sec.fission}}\right] \tag{2}$$

where: T is the time after the fission event in days.

3. BETA AND GAMMA ENERGY RELEASES

We consider the mean energies of the beta and gamma particles as:

$$\begin{aligned}\bar{E}_{\beta} &= 0.4\, MeV,\\ \bar{E}_{\gamma} &= 0.7\, MeV.\end{aligned}$$

where the average energy of the gammas is about twice that of the betas.

This suggests that the energy release from the beta emissions slightly exceeds the energy release from the gamma emissions from the fission products.

The rate of emission of beta and gamma ray energy can be written as:

$$\begin{aligned}\dot{E}(T) &= R_{\beta}(T)\bar{E}_{\beta} + R_{\gamma}(T)\bar{E}_{\gamma}\\ &= [\left(3.8\times 10^{-6}\times 0.4\right) + \left(1.9\times 10^{-6}\times 0.7\right)]T^{-1.2}\\ &= [\left(1.52\times 10^{-6}\right) + \left(1.33\times 10^{-6}\right)]T^{-1.2}\\ &= 2.85\times 10^{-6} T^{-1.2}\left[\frac{\text{MeV}}{\text{sec.fission}}\right]\end{aligned} \tag{3}$$

These simple empirical expressions are accurate within a factor of 2 or less [2].

4. DECAY POWER AFTER SHUTDOWN

If we want to take into consideration the operating time of a given core, and calculate the heat release after shutdown, we consider the time scale shown in Fig. 2 and adopt the following analysis.

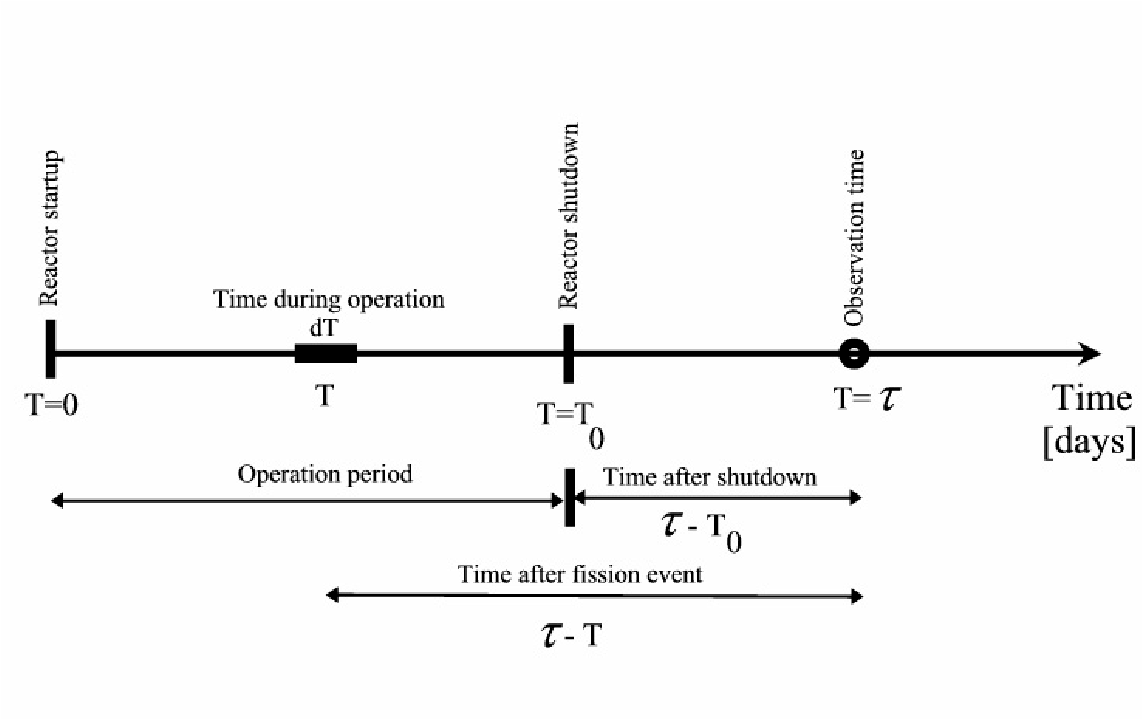

Figure2. Time sequence after reactor shutdown for decay heat calculation. Reactor power is P_0 in Watts.

The energy produced by fissions in the interval dT at time τ is:

$$\dot{E}_{\tau}(T) = 2.85 \times 10^{-6} (\tau - T)^{-1.2} [\frac{\text{MeV}}{\text{sec.fission}}] \quad (4)$$

For a reactor operating at a power of P thermal Watts (Wth), and considering that the energy release per fission event is 200 MeV of which 10 MeV are carried away by the antineutrinos associated with process negative beta decay, leading to a recoverable energy per fission event of 190 MeV / fission, one can write for the fission rate dF/dt:

$$\frac{dF}{dt} = P\ [\text{MWth}]\ .10^6 \left[\frac{\text{Watt}}{\text{MWth}}\right] [\frac{(\text{Joules/sec})}{\text{Watt}}] . \frac{1}{190} [\frac{\text{fission}}{\text{MeV}}] . \frac{1}{1.6 \text{ x } 10^{-13}} [\frac{\text{MeV}}{\text{Joule}}] .86{,}400 [\frac{\text{sec}}{\text{day}}]$$

$$= 2.84 \times 10^{21}\ P\ [\frac{\text{fissions}}{\text{day}}]$$

Notice that if we consider that fission's recoverable energy is 200 MeV, we get instead a value of 2.69 x 10^{21} fissions per day per MWth of power, a difference of 5 percent in the estimate. Some sources use the 200 MeV value and hence underestimate the decay heat power generation by 5 percent.

This leads to the equivalence:

$$2.84\times 10^{21}\left[\frac{fissions}{day}\right]\Leftrightarrow 1\ MWth\ of\ \ power$$

We can deduce that the number of fissions occurring in the interval dT is:

$$N = 2.84\times 10^{21} P_0 dT\ \ [\,fissions] \qquad (5)$$

where: P_0 is the reactor power level in MWth,
dT is an interval of time in days.

Thus the rate of emission of beta and gamma energy at time τ , due to fissions in the interval dT is:

$$\begin{aligned} R(T) &= \dot{E}_{\tau}(T).N \\ &= 2.85\times 10^{-6}(\tau - T)^{-1.2}[\frac{\text{MeV}}{\text{sec.fission}}]\times 2.84x10^{21} P_0 dT\ \ [\text{fissions}] \qquad (6) \\ &= 8.094\times 10^{15} P_0 (\tau - T)^{-1.2} dT[\frac{\text{MeV}}{\text{sec}}] \end{aligned}$$

We integrate over the irradiation period from "zero time" to "time T_0" the reactor shut down time to get:

$$\begin{aligned} P &= 8.094\times 10^{15} P_0 \int_{T=0}^{T=T_0} (\tau - T)^{-1.2} dT \\ &= 8.094\times 10^{15} P_0 \left[\frac{(\tau - T)^{-1.2+1}}{+0.2}\right]_0^{T_0} [\frac{\text{MeV}}{\text{sec}}] \end{aligned} \qquad (7)$$

Substituting the lower and upper limits:

$$\frac{P\left[\frac{MeV}{\text{sec}}\right]}{P_0\left[MWth\right]} = 4.05\times 10^{16}[(\tau - T_0)^{-0.2} - \tau^{-0.2}] \qquad (8)$$

Using the power in MWth units,

$$\frac{P\left[MWth\right]}{P_0\left[MWth\right]} = 4.05\times10^{16}\times1.6\times10^{-13}\,\frac{\text{MeV}}{\text{sec}}[(\tau-T_0)^{-0.2}-\tau^{-0.2}]\frac{\text{Watt.sec}}{\text{MeV}}10^{-6}\,\frac{MWth}{Watt} \quad (9)$$
$$= 6.48\times10^{-3}[(\tau-T_0)^{-0.2}-\tau^{-0.2}]$$

Notice that the time after shutdown is t = $(\tau - T_0)$, and the reactor operation time is T_0. Figure 3 shows the decay power percentage of the total power as a function of time for a typical reactor.

In term of the time after shutdown t:

$$P(t) = 6.48\times10^{-3}P_0[t^{-0.2}-(t+T_0)^{-0.2}] \;\; [MWth] \quad (9)'$$

EXAMPLE

At 1 second after shutdown for a reactor that operated for one year the decay power ratio would be:

$$\frac{P(t)}{P_0} = 6.48\times10^{-3}[t^{-0.2}-(t+T_0)^{-0.2}]$$
$$= 6.48\times10^{-3}[(\frac{1}{24\times60\times60})^{-0.2}-(\frac{1}{24\times60\times60}+365)^{-0.2}]$$
$$= 6.48\times10^{-3}[(0.0000157)^{-0.2}-(365.0000157)^{-0.2}]$$
$$= 6.48\times10^{-3}[9.13734-0.30729]$$
$$= 6.48\times10^{-3}\times8.83005$$
$$= 57.218\times10^{-3}$$
$$= 0.057218$$
$$\approx 6\ \%$$

At 1 minute after shutdown for a reactor that operated for one year the decay power ratio would be:

$$
\begin{aligned}
\frac{P(t)}{P_0} &= 6.48\times10^{-3}[t^{-0.2}-(t+T_0)^{-0.2}] \\
&= 6.48\times10^{-3}[(\frac{1}{24\times60})^{-0.2}-(\frac{1}{24\times60}+365)^{-0.2}] \\
&= 6.48\times10^{-3}[(0.000694)^{-0.2}-(365.000694)^{-0.2}] \\
&= 6.48\times10^{-3}[4.28280-0.30728] \\
&= 6.48\times10^{-3}\times4.52072 \\
&= 29.29\times10^{-3} \\
&= 0.02929 \\
&\approx 3\%
\end{aligned}
$$

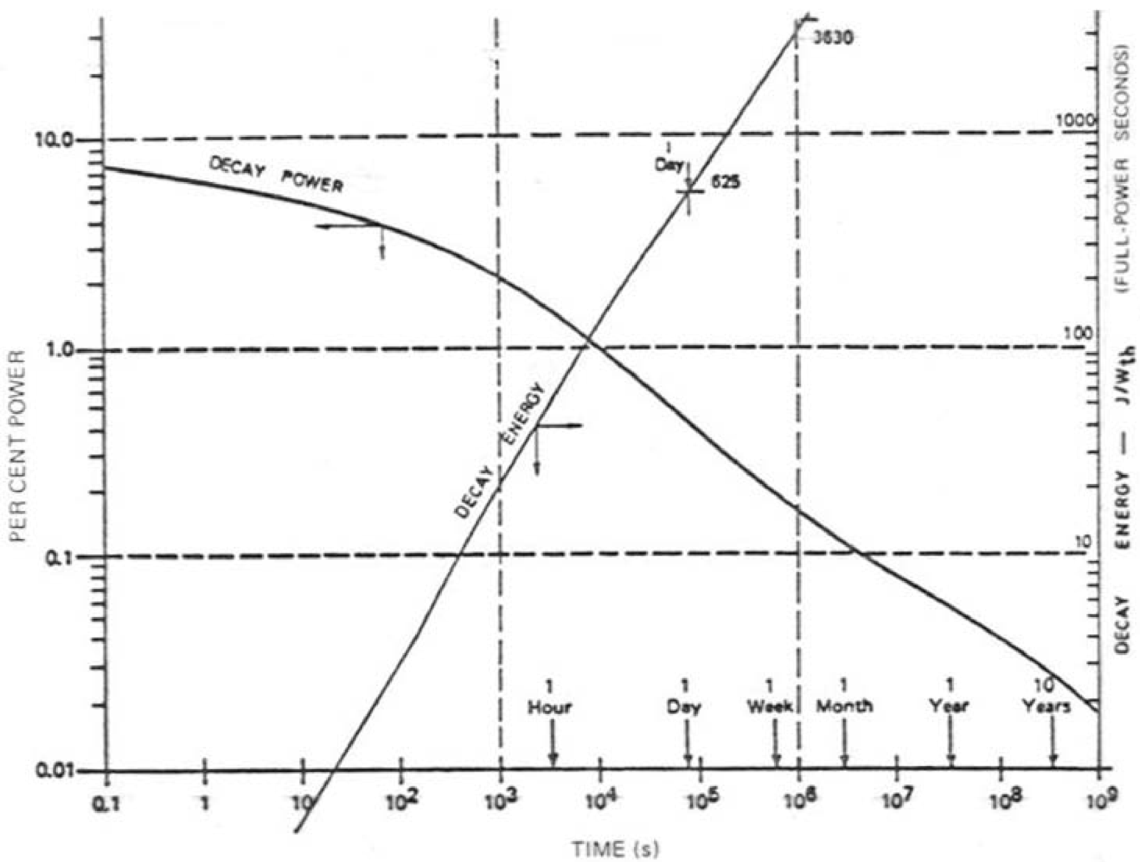

Figure 3. Decay heat power and energy release after shutdown as a function of time.

5. TOTAL HEAT GENERATION AFTER SHUTDOWN

An expression for the total energy release from decay heat generation after reactor shutdown can be derived by use of Eqn. 9. Let us consider the variable:

$$t = time\ \ after\ \ shutdown = (\tau - T_0)$$

thus the energy release after shutdown can be written as:

$$E(t) = \int_0^t P(t)dt\,[MWth.day] \tag{10}$$

By substituting from Eqn. 9 for the power P, we get:

$$E(t) = 6.48 \times 10^{-3} P_0 \int_0^t [(\tau - T_0)^{-0.2} - \tau^{-0.2}]dt$$

Substituting for:

$$t = (\tau - T_0)$$

we get:

$$E(t) = 6.48 \times 10^{-3} P_0 \int_0^t [t^{-0.2} - (t + T_0)^{-0.2}]dt$$

Carrying out the integration yields:

$$E(t) = 8.1 \times 10^{-3} P_0 [t^{0.8} - (t + T_0)^{0.8} + T_0^{0.8}] \tag{11}$$

where t is the time after shut down, and T_0 is the reactor operation time. This relationship is shown in Fig. 3.

A procedure is shown for the estimation of the decay heat power and energy release after shutdown for a typical Pressurized Water Reactor (PWR), as well as the decay heat power and energy release for a 3,000 MWth PWR with different operational times.

```
!       decayheat.f90
!       Procedure generating the decay heat power and integrated energy.
!       release for a Pressurized Water Reactor (PWR)
!       Decay heat power and integrated energy release after shutdown
!       for a constant reactor power P0, and for different operational
!       times t0.
!       Program saves output to file : toutput
!       This output file can be exported to a plotting routine, e.g. Excel
!       M. Ragheb, Univ. of Illinois at Urbana-Champaign

        program decayheat
        real t(11),lt(11),t0(4)
        real power(4,11),energy(4,11)
        real lpower(4,11),lenergy(4,11)
!       P0 is steady state reactor thermal power in MWth
```

```
        real :: p0=3000.0
!       Initialize time scale in seconds
        tttt=1./(24.*60.*60.)
!       Generate time scale ticks
        t(1)=0.1*tttt
        write(*,*) t(1)
        do i=2,11
                t(i)=t(i-1)*10.0
                write (*,*) t(i)
        end do
        pause
!       Open output file for Excel plotting
        open(unit=10,file='toutput.xls')
!       Define exponents in decay heat formula
        xx=-0.2
        x=0.8
!       Initialize initial reactor operating times T0 (days)
!       T0=1 week
        t0(1)=7.0
!       T0=1 month
        t0(2)=30.
!       T0=1 year
        t0(3)=365.0
!       T0= 10 years
        t0(4)=3650.0
                do i=1,4
                        do j=1,11
!       t is time after shutdown (days)
                        tt=t(j)+t0(i)
!       power is decay power, lpower is natural logarithm of decay power
                        power(i,j)=6.48e-03*p0*((t(j)**xx)-(tt**xx))
                        lpower(i,j)=log(power(i,j))
!                       write (*,*)lpower(i,j)
!       energy is integral decay heat energy release
!       lenergy is natural logarithm of decay energy
                        energy(i,j)=7.60e-03*p0*((t(j)**x)-(tt**x)+(t0(i)**x))
                        lenergy(i,j)=log(energy(i,j))
!       calculate natural logarithm of energy
                        lt(j)=log(t(j))
                write(*,*)lt(j),lpower(i,j),lenergy(i,j)
                        end do
                end do
!       Write results on output file
                do j=1,11
                        write(10,100)lt(j),(lpower(i,j),lenergy(i,j), i=1,4)
100     format(9(E11.5,1x))
                end do
        write(*,*)'End of program run, Please enter return to continue'
        pause
        stop
        end
```

Figure 4. Procedure for the estimation of the decay heat power and energy release.

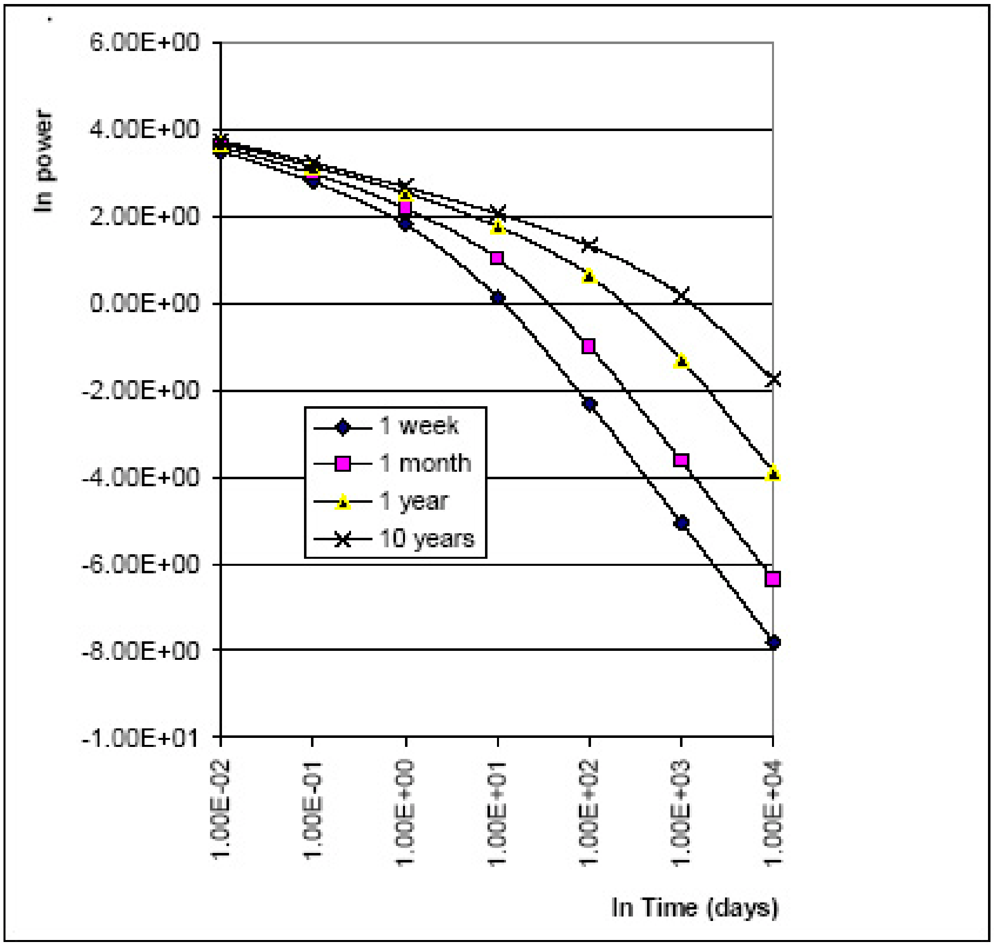

Figure 5. Decay heat power release for a 3,000 MWth PWR for different operational times.

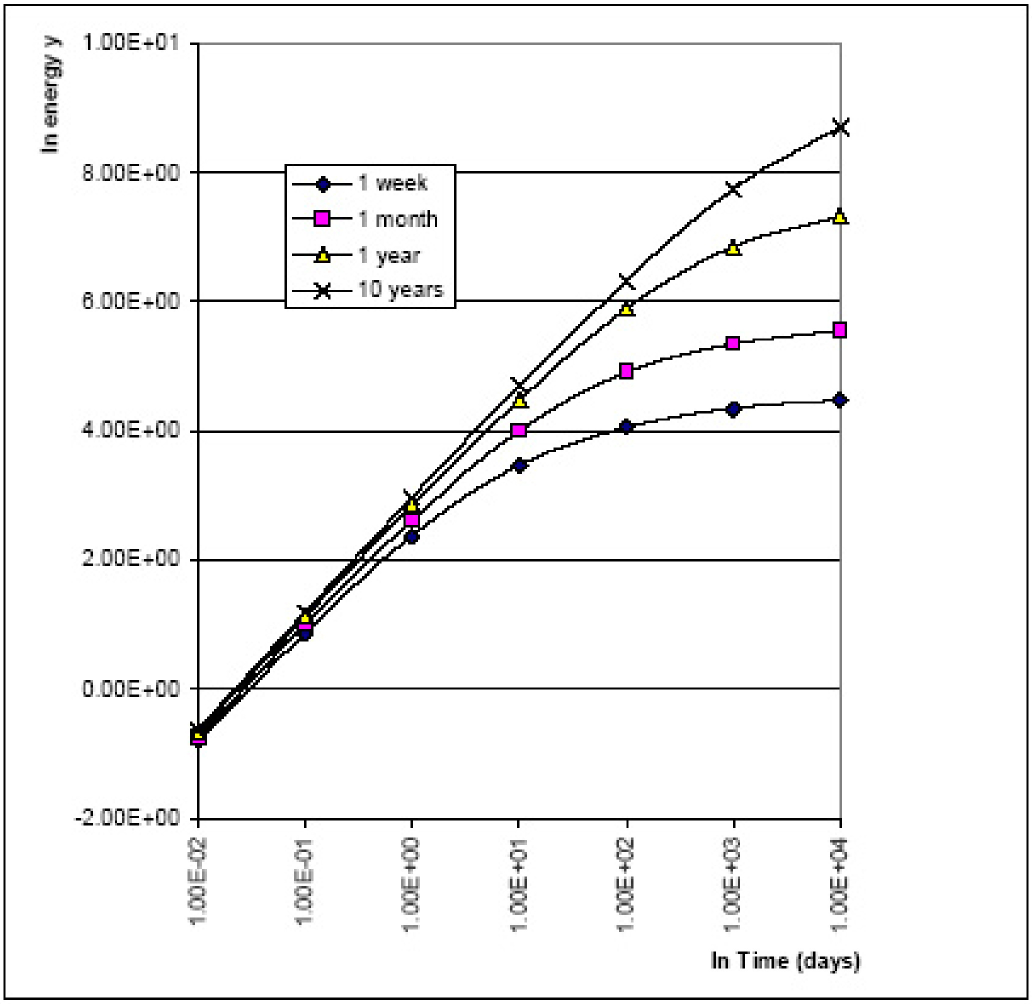

Figure 6. Decay energy release after shutdown for a 3,000 MWth PWR for different operational times.

6. SAFETY IMPLICATIONS

If the decay heat is not successfully extracted after a core shutdown, damage to the core in terms of fuel damage, hydrogen production, melting or even vaporization would occur. This unique feature characterizes nuclear power plants from other heat engines, which would stop heat generation once their operation is stopped.

The misunderstanding of this particular feature of nuclear power plants has led uninformed operators to commit serious human errors partially contributing to serious reactor accidents such as the Three Mile Island accident.

For instance, if one considers a reactor operation time of T_0 = 1 year, and consider a reactor power of P_0 = 3,000 MWth, substitution in Eqn. 11 yields for the energy release one day after shutdown:

$$E(1day) = 7.6\times10^{-3}\times3,000[1^{0.8}-(1+365)^{0.8}+365^{0.8}]$$
$$=22.8[1^{0.8}-(366)^{0.8}+365^{0.8}] \quad (12)$$
$$=17.20[MWth.day]$$

and for 1 month after shutdown:

$$E(1month) = 22.8[30^{0.8}-(395)^{0.8}+365^{0.8}]$$
$$=179.64[MWth.day] \quad (13)$$

These are substantial amounts of energy release. As shown in Fig. 4, this heat release leads to the rise of the reactor temperature. In the hypothetical adiabatic heating accident, where no heat extraction is assumed, the rise in reactor temperature would eventually lead to melting of the fuel (Fig.5) and a subsequent fission product release from the central void in the fuel pellets and results in the danger of a plant hazard.

If this were a feature of all nuclear reactor designs, then it may be argued that the risk involved can be justifiably borne to enjoy the benefit of nuclear electricity. However, this is not necessarily the case, since nuclear reactors can be designed to be forgiving and inherently safe.

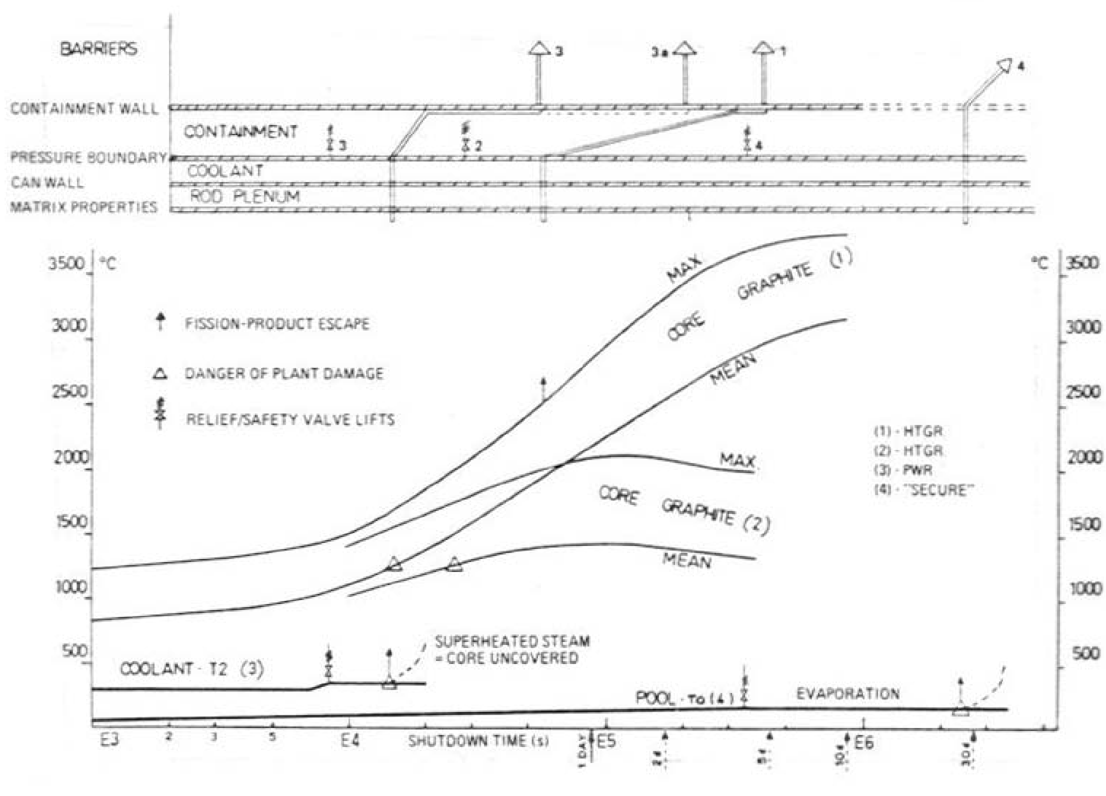

Figure 7. Consequences of unrestricted core heat up in different reactor systems. HTGR: High Temperature Gas-cooled Reactor, PWR: Pressirized Water Reactor.

Typical operational temperatures that should not be exceeded to avoid fission products release are shown in Table 1.

Table 1. Operational temperature of different reactor fuels.

Reactor concept	**Temperature Degrees Celsius**
Magnesium alloy cladding (Magnox)	450
AGR stainless steel cladding	750
Boiling Water reactor (BWR)	300
Pressurized Water Reactor (PWR)	320
Liquid Metal Reactor (LMR), Na cooled	750

The time at which damage occurs is particularly short in a PWR system at about 1.5×10^4 seconds = 4.2 hours, compared with the High Temperature Gas Cooled Reactor (HTGR) or the district heating low temperature SECURE system which can be extended to 30 days.

Systems that can recover from an increase in core temperature in an accident situation are designated as “inherently safe,” forgiving,” or “passive” reactor designs, and are an active area of safety research and development. They ought be adopted as replacements of aging earlier design power plants instead of extending their operational lives.

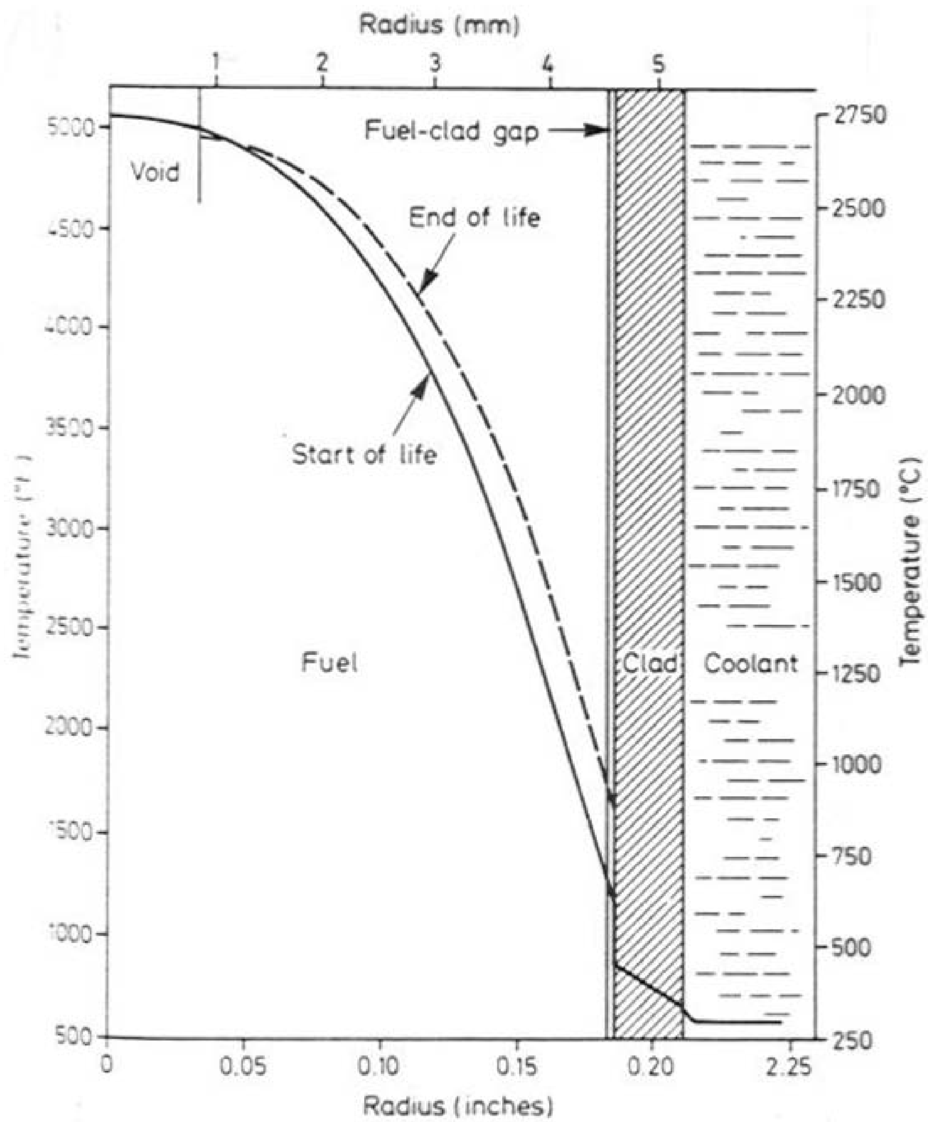

Figure 8. Fuel element temperature profile for PWR fuel.

7. DECAY HEAT POWER AND INTEGRATED POWER FROM THE SYSTEM ANALYSIS HANDBOOK

The decay heat power ration following the shutdown of a reactor is shown in Fig. 9. A reactor that operated for a period of 10^{13} seconds is considered to have operated for an infinite time.

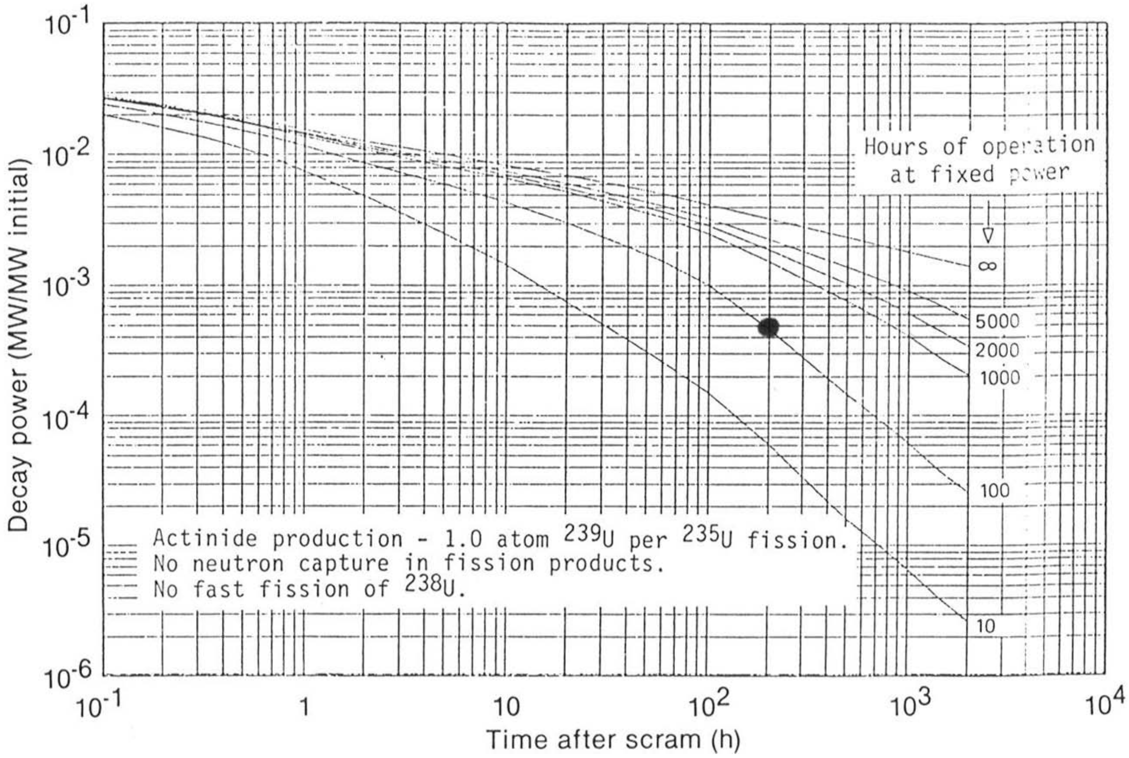

Figure 9. Decay heat power ratio for U^{235} fuel as a function of time after shutdown.

The energy release after shutdown for different periods of operation is shown in Fig. 9. The graphs are based on the ANSI / ANS 5.1 standard of 1979. The decay heat power involves the combined release from U^{235} fission products and actinides U^{239} and Np^{239} decay. The constants for the actinides and 23 fission groups from the standard are incorporated into the thermal hydraulics code RELAP5 / MOD 1.6 to obtain the decay power and energy release.

The data do not include the contribution from the thermal fission of Pu^{239} or the fast fission of U^{238}, which would add about 2 percent to the power for light water reactors, but may be substantial in a fast reactor spectrum. The effect of fuel burnup on the fission products is not included. The fission energy release from delayed neutrons fission depends on the negative reactivity at shutdown and could account for 1-2 MWth.sec per MWth of initial reactor power.

Both the decay heat and energy release for an arbitrary power history can be estimated by linearizing the power history into time intervals of constant power levels, computing the contributions from each time interval and summing them to obtain the total values.

EXAMPLE

Consider a reactor that operated for 100 hours at a steady power level of 1,000 MWth, then shut down for 80 hours, and then operated again for 100 hours at a steady state power of 500 MWth, before being shut down. We wish to estimate the decay heat power 20 hours after shutdown.

The first operating interval contributes:

$$P_1 = 1,000[MWth] \times 4.5 \times 10^{-4} [\frac{MWth}{MWth}] \tag{14}$$

read from Fig. 9 at 100 hours operation at 1,000 MWth, and at 200 hours after the first shutdown.

The second operating interval contributes the following decay power:

$$P_2 = 500[MWth] \times 3.00 \times 10^{-3} [\frac{MWth}{MWth}] \tag{15}$$

read at 100 hours operation at 500 MWth and 20 hours after shutdown.

Adding the contributions from the two operational periods from Eqns. 14 and 15, we get:

$$\begin{aligned} P_{total} &= P_1 + P_2 \\ &= 1,000[MWth] \times 0.00045 [\frac{MWth}{MWth}] + 500[MWth] \times 0.003 [\frac{MWth}{MWth}] \\ &= 0.45 + 1.5 \\ &= 1.95\ MWth \end{aligned} \tag{16}$$

Notice that the contribution from the second operational interval is larger than from the first one. The decay heat generation 20 hours after shutdown is a small fraction of the operational power level.

Correction factors accounting for neutron capture in the fission products, and actinide correction factors can be optionally applied if the production rate of U^{239} / U^{235} fissions is available.

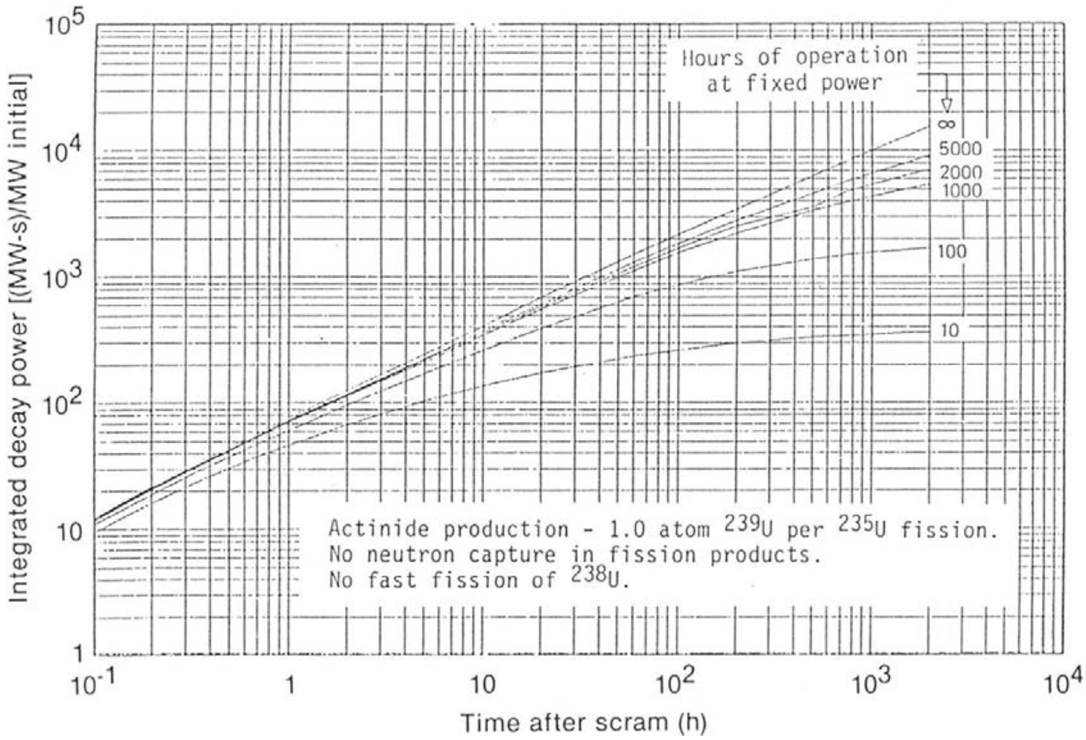

Figure 10. Decay heat energy release for U^{235} fuel as a function of time after shutdown.

7. OTHER REPRESENTATIONS

Way and Wigner derived and expression for the beta and gamma energy from the fission products as:

$$\frac{P}{P_0} = 6.22 \times 10^{-3} t^{-1.2} \tag{17}$$

where t is the time after irradiation in seconds.

The estimated total decay heat power is double the value in Eqn. 12.

A more accurate representation of the decay heat power for short cooling times is given by the empirical equation [7]:

$$\frac{P}{P_0} = 0.1\{(t+10)^{-0.2} - (t+T_0+10)^{-0.2} - 0.87[(t+2x10^7)^{-0.2} - (t+T_0+2x10^7)^{-0.2}]\} \tag{18}$$

where the time now is in seconds instead of days. This expression includes an allowance for the heat produced by the beta decay of U^{239} and Np^{239} resulting from the radiative capture of neutrons in U^{238}.

The heat generated from the U^{235} fission products alone can be obtained by subtracting the approximate expressions for the heat generated by the decay of U^{239} and Np^{239} as:

$$\frac{P_{U^{239}}}{P_0} = 0.0025[e^{-(\frac{t}{2,040})} - e^{-(\frac{t+T_0}{2,040})}]$$
$$\frac{P_{Np^{239}}}{P_0} = 0.0013[e^{-(\frac{t}{290,000})} - e^{-(\frac{t+T_0}{290,000})}] \quad (19)$$

8. DISCUSSION

The decay heat generation decreases rapidly after shutdown and becomes a small fraction of the operational fission power level.

However, the safety design of a nuclear power plant must include provisions in terms of pumping facilities and decay heat exchange equipment that could accommodate the decay heat generation immediately after shutdown, which would amount to about 6 percent of the operational power level at one second after shutdown.

EXERCISES

1. Prove that 1 Watt(th) of power corfresponds to 2.84×10^{15} [fissions/day], starting from the consideration that the fission of an atom of uranium releases 190{MeV/fission] of enrgy, or from the burning of about 1.112 gms of U^{235} per day generates a power output of 1 MWth.
2. Modify the computer procedure listed in the Appendix to generate plots of the decay heat power and integral energy release for a 1,500 MWth reactor that operated for a period of 1 year.
Compare the results to those of a reactor that operated for 10 years.
3. A reactor has the following power history:
a) Operation at a power level of 3,000 MWth for 1 year.
b) Operation at a power level of 2,000 MWth for 6 months, followed by a scram.
Determine the decay heat power at the following times using the Systems Analysis Handbook data:
    i) Six minutes after shutdown.
    ii) One day after shutdown,
    iii) One month after shutdown.
4. The relation for the decay heat power versus time P(t) from the fission products assuming an infinite irradiation period is given in the reference: "Decay Heat Power in Light Water Reactors," ANSI/ANS-5.1, published by the American Nuclear Society (ANS) as:

$$\frac{P(t)}{P_0} = At^{-a}$$

where t is the time after shutdown in seconds., and:

A = 0.0603, a= 0.0639 for $0<t<10$ s

$A = 0.0766$, $a = 0.1810$ for $10<t<150$ s

$A = 0.1300$, $a = 0.2830$ for $150<t<4 \times 10^6$ s.

1. Derive an expression for the total energy release between the times t_1 and t_2.
2. For a power reactor producing $P_0 = 3{,}000$ MWth, calculate the total energy release from the decay heat within 10 second, 150 seconds and 4×10^6 seconds after shutdown in MegaJoules (MJs).

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