Thermo-Mechanical Fatigue Life Prediction:
A Critical Review

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ABSTRACT

Improved prediction methods for thermo-mechanical fatigue life will assist in reducing life cycle costs and increasing the availability of the hot-section components in aircraft engines. Literature on thermo-mechanical fatigue life assessment is reviewed in this report, with an emphasis on the life prediction models applied in aircraft engines. Successful areas of application of these life prediction models are addressed as well as their limitations. Published quantitative thermo-mechanical fatigue life data for selected hot section materials is also summarised. The review concludes by indicating areas where knowledge is deficient and where further research would be most beneficial.

RELEASE LIMITATION

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Executive Summary

Aircraft engine hot-section components are designed to operate in high-temperature environments, with high thermal gradients and mechanical loads. Repeated engine start-up and shut-down operations subject components to cyclic strains which are generated both thermally and mechanically. These thermo-mechanical fatigue cycles cause microstructural material damage to the components, and lead ultimately to fatigue crack initiation and failure. Advanced high temperature fatigue life prediction methodologies are needed to decide when to replace engine components, in order to increase reliability and service life, and reduce maintenance costs.

Predicting the lives of components subject to thermo-mechanical fatigue continues to be a great challenge. In the last four decades much work has been performed in modelling fatigue crack initiation for a range of materials. This report gives an overview of the numerous thermo-mechanical fatigue life prediction models developed to date. The more popular models currently used to predict crack initiation lives are described in detail.

The popular models for thermo-mechanical fatigue crack initiation are the Damage Summation (DS), Frequency Separation (FS), Ductility Exhaustion (DE), Strain-Range Partitioning (SRP), Total-Strain Version of SRP (TS-SRP), and Strain Energy Partitioning (SEP). The life equations for each model are given, along with descriptions of some of the validating tests which have been conducted. Each model has been found to be useful for particular categories of material over specific domains of temperature and cyclic strain, and the review indicates the applicability of the different life prediction models. No model has been found which invariably gives better predictions than the others, so the choice of a model must be based on the knowledge that it has been validated in the relevant circumstances. All the models have some limits on accuracy within the domains where they have been validated, and greater accuracy limitations beyond. The review also indicates the areas where further research is most likely to improve life prediction methods.

Published quantitative data on the thermo-mechanical fatigue life characteristics for gas turbine hot-section materials are very limited. An attempt has been made to collate some of the data that are available in the literature. These data are given in the Appendix, providing a useful database for life assessment of engine hot-section components.
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Mr Neil Swansson is head of Engine Mechanical Integrity in the Airframes & Engines Division at AMRL. He graduated B.Mech.E with first class honours from the University of Melbourne in 1955, and then spent 2 years with Rolls Royce Ltd, Aero Engine Division in UK. In 1957 he joined Australian Defence at ATEA (then Army Design Establishment), spending some 5 years designing military vehicle transmissions. He transferred to AMRL in 1962 and has since investigated most mechanical aspects and problems of aircraft turbine engines, and has produced some 35 publications, reports, conference papers &c. His research areas include strength and failure analysis of engine components, vibration of engines and rotor systems, and failure monitoring of gears and bearings. To investigate life management of aircraft engine components subject to low cycle fatigue, an area of concern to defence in the 1980s, he spent 15 months at the Naval Air Development Center in Pennsylvania, USA, in 1986. Recently attention has turned toward the engine hot section where high maintenance costs are incurred, and he has initiated new programs including high temperature structural analysis, temperature modelling and heat transfer in turbine engines.
Nomenclature

A  general constant
   a exponent on strain energy partitioning equations
B  intercept of elastic strain-range vs. life relations
   b exponent of cyclic life for elastic strain-range vs. life relations
C  intercept of inelastic strain-range vs. life relations
   C’ intercept of equivalent inelastic line for combined creep-life cycles
   c exponent of cyclic life for inelastic strain-range vs. life relations
D  ductility, or damage
E  Young’s modulus
F  strain fraction
K  cyclic strain-hardening coefficient
   k exponent on frequency
m  general exponent of time in empirical flow equations
N  applied cycles in prediction, or number of cycles to failure
n  cyclic strain hardening exponent
T  temperature, or total time to rupture
   t hold time, sec
U  strain energy
   α exponent on Coffin-Manson relationship
   β exponent on inelastic strain-range
   Δ range of variable
   ε strain
   ν frequency
   σ stress
   τ time

Subscripts:
   c  pure creep, or compression-going
   cc tensile creep reversed by compressive creep
   cp tensile creep reversed by compressive plasticity
   el elastic
   f  failure
   i  index for specific fatigue conditions
   in inelastic
   ij pp, cc, pc, cp
   p  pure fatigue
   pc tensile plasticity reversed by compressive creep
   pp tensile plasticity reversed by compressive plasticity
   t  tension-going
1. Introduction

Reliable life assessment of engine structural components using the state-of-the-art life prediction models, forms the basis of good design and of design modifications which may lead to component life extension. In aircraft gas turbine engines, hot section components such as combustor liners, turbine blades, disks and nozzle guide vanes, are designed to operate in high temperature environments with high thermal gradients. They are subject to cyclic strains that are induced thermally as well as mechanically. Under these cyclic temperatures and strains, thermo-mechanical fatigue (TMF) damage occurs in the material, leading to initiation of cracking and subsequent crack growth. Generally life prediction for turbine blades is based solely on crack initiation while for vanes and combustor liners, which have much greater damage tolerance, crack growth life may be used as well as initiation life (Viswanathan 1989).

During repeated start-ups and shut-downs, engine hot section components experience cyclic strains, which are induced both thermally by rapid gas temperature changes, and mechanically by centrifugal force and pressure difference. In the last four decades significant improvements have been made in the reliability and durability of these components. However predicting the stress and strain response of components and hence their lives under this TMF loading is a great challenge. At the microstructural level, damage processes occurring under TMF are more complex than under isothermal fatigue. Sophisticated life prediction models have been developed to take creep-fatigue interactions and other damage effects into account, and so improve life predictions under TMF conditions. Much work has been performed, with qualified success, in modelling the TMF life of materials on a crack initiation basis.

This report gives a survey of TMF modelling methodologies for crack initiation developed to date, and ongoing developments in the area are described. It also evaluates the applicability of the different life prediction models. Fracture mechanics models for crack growth prediction are not included. Published data on the TMF life characteristics for gas turbine hot-section materials are very limited. To establish a useful database for the life assessment of engine hot-section components, selected data available in the literature have been collated and are given in the Appendix.

During the literature survey, it was found that there are numerous modelling techniques available. For TMF crack initiation, more than one hundred life prediction approaches and variations exist (Halford 1993). Since a large number of models have been proposed and developed, this review does not attempt cover all of them. Only the more popular and advanced modelling techniques are chosen and addressed. Most of the models once proposed have not been followed up. The popular models for TMF crack initiation described in this review are the Damage Summation (DS), Frequency Separation (FS), Ductility Exhaustion (DE), Strain-Range Partitioning (SRP), Total-Strain Version of SRP (TS-SRP), and Strain Energy Partitioning (SEP). No one particular model has always been found to give more accurate predictions than the others. The review not only presents the merits of the models, but also examines their limitations.
2. Thermo-Mechanical Fatigue

Thermo-mechanical fatigue is the synergistic damage process caused by cyclic thermal and mechanical loading, and the theory of TMF addresses the creep-fatigue interactions that occur. When hot-section components are subject to high temperature thermal cycles concurrently with mechanical strain cycles, TMF conditions resulting in microstructural damage occur. The lifetime of the components under such TMF loading is found to be quite different from that obtained in isothermal low-cycle fatigue tests conducted at the maximum temperature of operation, where creep damage of the material would be greatest. Thomes et al. (1982) indicated that at the same strain-range, the creep-fatigue damage of Inconel 738 material at temperature cycles from 500 to 850 °C was more severe than that under isothermal testing at 850 °C. Furthermore, for the nickel-based superalloy MAR-M-200, Bill et al. (1984) found that the lives of material specimens under in-phase thermo-mechanical cycling at temperatures from 495 to 1000 °C were well below isothermal creep-fatigue lives obtained at temperatures of 650, 927, and 1000 °C. They concluded that the lifetimes for actual gas turbine components should be predicted using TMF methods rather than methods for isothermal conditions.

When both thermal and mechanical cycles vary in an arbitrary manner, predicting lives becomes very difficult. Modern laboratory testing techniques allow realistic simulation of cycle shapes representing actual service conditions. There are two extreme modes; in-phase and out-of-phase cycles, as shown in Figure 1. Under in-phase cyclic loading conditions, the maximum tensile strain occurs at the same time as the peak temperature, and the maximum compressive strain occurs at the minimum temperature. In contrast, under out-of-phase cycles, the maximum tensile strain occurs at the minimum temperature, and maximum compressive strain at the peak temperature. The phasing effect on TMF life is variable and hard to generalise, and will be not covered in this review. Detailed description of phase effects can be found in the paper of Halford (1987). TMF phase effect is dependent upon the alloy under study, temperature levels, and cycle times. Also the phasing effect on TMF life differs in the cases of crack initiation and crack growth.

Accurate prediction of TMF lives of engine hot-section components, needs sophisticated life modelling techniques. A single damage mechanism is not adequate for characterising all TMF damage, therefore a number of damage mechanisms and damage models have been proposed. TMF life prediction may include both TMF crack initiation models and crack propagation models, but as previously noted only the former are addressed in the following sections.
Figure 1. In-phase and Out-of-phase TMF loading patterns

The more sophisticated models require many variables and associated parameters in the life equations, to represent the principal damage mechanisms. Stabilised values of the variables are used in the life equations, since when cyclic loading is first applied, a number of shakedown cycles are often needed to stabilise conditions. The variables include elastic, inelastic, and total strain-ranges, dissipated strain energy, temperature, frequency, hold time, strain rate, and mean stress (Chaboche 1982).

### 3. Life Prediction Methodologies

Thermo-mechanical fatigue life prediction models take into account the interaction between fatigue and creep at varying temperatures. The difficulty in understanding and predicting the interaction behaviour has stimulated significant research effort in finding good life prediction models. The models used include damage-based criteria, stress-based criteria, strain-based criteria and energy-based criteria. The following review focuses on the more popular and advanced TMF models, comprising the Damage Summation (DS), Frequency Separation (FS), Ductility Exhaustion (DE), Strain-Range Partitioning (SRP), Total Strain Version of SRP (TS-SRP), and Strain Energy Partitioning (SEP).

#### 3.1 Damage Summation Model

The linear Damage Summation (DS) model (also called the linear life fraction, or linear cumulative damage) is the simplest expression for creep-fatigue life prediction. It ignores the microstructural details of the damage process. It was introduced first by Taira (1962) as follows:
\[ D_{\text{fatigue}} + D_{\text{creep}} = D_{\text{total}} \]  \hspace{1cm} (1)

\( D_{\text{total}} \) is the total damage, \( D_{\text{fatigue}} \) and \( D_{\text{creep}} \) are the fatigue damage and creep damage respectively. The damage summation rule assumes that failure occurs when the sum of the fatigue damage and the creep damage is equal to a critical value. By means of the Miner rule (1945) for fatigue damage and Robinson rule (1952) for creep damage, with repeated application of a single simple cycle, Equation (1) becomes:

\[ N_i \left( \frac{1}{N_p} + \frac{\tau}{T_c} \right) = D_{\text{total}} \]  \hspace{1cm} (2)

\( N_i \) is the number of cycles to failure at a given strain-range, \( N_p \) is the pure fatigue life at that strain-range. The creep damage fraction in each cycle is \( \tau / T_c \), in which \( \tau \) is the hold time at a given stress in a cycle and \( T_c \) is the time to rupture under static creep at that stress.

Under thermal-mechanical strain cycles, the creep damage consists both of a loading creep strain (as load/temperature is held constant or increases with time above a threshold creep value) and dwell creep strain (when strain is held constant over the hold time resulting in stress relaxation). Ellison (1994) modified life prediction by this fraction rule to deal with the changing strain, summing over \( \tau / T_c \) (approximated as a series of steps) as follows:

\[ \left( \frac{\tau}{T_c} \right)_{\text{total}} = \sum \left( \frac{\tau_i}{T_{c_i}} \right)_{\text{loading}} + \sum \left( \frac{\tau_i}{T_{c_i}} \right)_{\text{dwell}} \]  \hspace{1cm} (3)

Analogously to summing the creep damage fraction, fatigue damage is obtained by summing the cyclic damage under differing loadings. The linear damage summation in Equation (2) should then be written as:

\[ \sum \frac{N_i}{N_p} + \sum \frac{\tau_i}{T_{c_i}} = D_{\text{total}} \]  \hspace{1cm} (4)

where the subscript \( i \) refers to specific fatigue conditions. In many cases, the DS model can adequately predict experimental behaviour for engineering applications, especially for thermal fatigue (Spera 1972). For AISI 304 and 316 stainless steel, the total damage \( D_{\text{total}} \) at failure is equal to unity (Del Puglia and Manfredi 1979). Because of the model’s simplicity, it has been adopted in the ASME Boiler and Pressure Vessel Code, Section III, Case N-47 (1974) to predict the creep-fatigue life (Mottot et al. 1982). However, the following factors should be borne in mind when applying the damage summation model.

1. The damage summation model ignores the failure process, in other words, the sequence effect on the creep-fatigue life is omitted. In this sense, it assumes that
the subsequent creep life after prior fatigue is the same as the subsequent fatigue life after prior creep. But in many cases, creep damage and fatigue damage are not independent. Tien, Nair and Nardone (1983) found that for cyclic softening materials prior fatigue causes small cracks, the presence of which will reduce the subsequent creep life, while in an austenitic stainless steel prior creep may enhance subsequent fatigue life.

2. Ellison and Zamily (1994) pointed out that the lives of some steels predicted by the damage summation model were not conservative. Bicego et al. (1988) indicated that non-conservative predictions seem to be associated with metallurgical inhomogeneities which lead to premature failures.

3. The creep rupture properties determined in monotonic tests used in the life prediction may not be appropriate to simulate the cyclical hardening or softening materials.

3.2 Frequency Separation Model

The frequency separation (FS) model incorporates the effect of cyclic frequency upon the creep fatigue life. Coffin (1976) extended the Coffin-Manson Law and postulated a power-law life relationship given in Equation (5) with three principal variables, inelastic strain-range \( \Delta \varepsilon_{\text{in}} \), tension-going pseudo-frequency \( \nu_t \), and cycle-time unbalance \( \nu_c / \nu_t \). \( \nu_t \) and \( \nu_c \) are reciprocals of the tension-going and compression-going times \( \tau_t \) and \( \tau_c \) respectively, as shown in Figure 2.

\[
N_f = C \Delta \varepsilon_{\text{in}}^\beta \nu_t^m (\nu_c / \nu_t)^k
\]

(5)

C, \( \beta \), m, and k are constants dependent on material, environment and temperature. They are determined by ‘slow-fast’ or ‘fast-slow’ tests.

The FS model was developed to deal properly with very complex wave shapes. The concept of tension-going damage is based on the microcrack propagation process, assuming that low-cycle fatigue damage can be measured by the microcrack growth. During the tension-going part of the cycle the microcrack opens and advances while during compression the crack closes and does not advance. This simple modification separating tension-going and compression-going damage provided a significant error reduction in high temperature low-cycle fatigue life prediction.
In developing the FS model, Coffin (1976) obtained good agreement between fatigue life predicted from the model and actual measured life in experiments, for type 316 stainless steel. Further, the FS model was able to predict all the verification tests, except for the slow-fast tests, to within a factor of two which is the practically realisable accuracy (Bernstein 1982). However, Del Puglia and Manfredi (1979) suggested that if the compressive part of the cycle influences the tensile half, the FS model may be invalid. Another drawback of the FS model indicated by Manson (1973) is that the frequency modification approach requires considerable input data. For each temperature of interest, four or more constants need to be determined from separate tests. Thus for complex TMF cyclic loading, the FS model is fairly cumbersome.

The frequency separation model in Equation (5) is the latest form in the development of a series of frequency modified models. These models have been proposed in various forms, such as the Frequency Modified model (Coffin 1970), and Ostergren models (1979). All these models use frequency as a principal variable.

### 3.3 Ductility Exhaustion Model

The Ductility Exhaustion (DE) Model derived originally from the Coffin-Manson relationship between the inelastic strain-range and the number of cycles to failure given by:

$$\Delta \varepsilon_{in} N_{f}^{\alpha} = D$$

(6)

The constant $\alpha$ is approximately $\frac{1}{2}$ at ambient temperature, but at elevated temperature it increases to near unity for many materials (Ellison 1969). The constant $D$ has been commonly related to the tensile ductility. From the research of creep behaviour of 2.25Cr-1Mo Steel, Edmunds and White (1966) confirmed that the total
accumulated creep strain in a dwell period is equal to the creep ductility of the material. So the predicted lifetime $N_c$ for creep dominated failure can be written as:

$$\Delta \varepsilon_c N_c = D_c$$  \hspace{1cm} (7)$$

$\Delta \varepsilon_c$ is the tensile creep strain per cycle, and $D_c$ is the low band creep ductility. Priest and Ellison (1981) developed and applied the above equation to predict the creep-fatigue life, by separating the total damage into two components, creep damage and fatigue damage. The creep damage is controlled by Equation (7), while fatigue damage is dominated by plastic strain given by:

$$\Delta \varepsilon_p N_p = D_p$$  \hspace{1cm} (8)$$

where $\Delta \varepsilon_p$ is the effective plastic strain, and $D_p$ is the fatigue ductility. By using a simple interactive creep-fatigue damage rule, the lifetime of creep-fatigue can then be calculated in the following form.

$$\frac{1}{N_f} = \frac{1}{N_c} + \frac{1}{N_p}$$  \hspace{1cm} (9)$$

To apply the DE model for life prediction, $N_c$ and $N_p$ have to be obtained. For the calculation of $N_p$ based on Equation (8), $\Delta \varepsilon_p$ is normally determined from the stabilised stress-strain hysteresis loop in the actual creep-fatigue test, and then $D_p$ is obtained from a pure fatigue test conducted to failure under the same plastic strain-range. For the calculation of $N_c$ based on Equation (7), $\Delta \varepsilon_c$ is also determined from the stabilised stress-strain hysteresis loop in the actual creep-fatigue test, but $D_c$ should be obtained after stress relaxation (or at the critical strain rate). Conduct of these tests is difficult, so it is not easy to select appropriate values for $D_c$.

The DE model is simple to use. It has been successfully used to interpret test data (Priest and Ellison 1981). Nevertheless, Coffin (1967) and Ellison (1994) indicated that the lifetime of creep-fatigue predicted by the DE model is normally conservative, and sometimes may be too conservative. This may possibly be caused by oxidation and/or specimen geometric instability.

### 3.4 Strain-Range Partitioning Model

For high temperature low-cycle fatigue, the failure process is affected by the time-dependent cyclic stress-strain behaviour over the entire temperature range. To account for the time dependent portions of the cycle, Manson et al. (1971) developed the strain-range partitioning (SRP) method. The SRP life prediction model partitions the inelastic strain-range into time-independent plasticity and time-dependent creep, rather than working with the total inelastic strain-range alone. Each component contributes a certain fraction to the total damage. Under cyclic reversed loading, there are four possible combination cycles of inelastic strain, which the SRP model needs to consider. The cycle types are: tensile plasticity reversed by compressive plasticity (PP), tensile...
creep reversed by compressive creep (CC), tensile creep reversed by compressive plasticity (CP), and tensile plasticity reversed by compressive creep (PC). In any stabilised combined cycle, a maximum of three cycle types are physically possible, PP, CC, and either PC or CP.

At first, the SRP model was based on the following simple linear damage rule (Manson, Halford and Hirschberg 1971).

\[
\frac{1}{N_f} = \frac{1}{N_{pp}} + \frac{1}{N_{cc}} + \frac{1}{N_{pc}} + \frac{1}{N_{cp}} \tag{10}
\]

\(N_{pp}, N_{cc}, N_{pc}\), and \(N_{cp}\) are the fatigue lives produced by PP, CC, PC and CP inelastic strain cycles respectively. The life data can be conveniently expressed in the classical Coffin-Manson form by Equations (13)-(16).

Two years later, Manson and Halford (Manson 1973) modified the SRP model by means of the interaction damage rule, which uses strain-range fractions to partition damage. The expression can be written according to Hoffelner et al. (1983)

\[
\frac{1}{N_f} = \frac{F_{pp}}{N'_{pp}} + \frac{F_{cc}}{N'_{cc}} + \frac{F_{pc}}{N'_{pc}} + \frac{F_{cp}}{N'_{cp}} \tag{11}
\]

\(F_{pp}, F_{cc}, F_{pc}\), and \(F_{cp}\) are PP, CC, PC and CP strain-range fractions, for example, \(F_{pp} = \Delta \varepsilon_{pp}/\Delta \varepsilon_{in}\). \(N'_{pp}, N'_{cc}, N'_{pc}\), and \(N'_{cp}\) are the fatigue lives produced by PP, CC, PC and CP inelastic strain cycles respectively, but they are calculated according to equations (13)-(16) using total inelastic strain range, \(\Delta \varepsilon_{in}\) rather than the partitioned inelastic strain-range components (\(\Delta \varepsilon_{pp}\), \(\Delta \varepsilon_{cc}\), \(\Delta \varepsilon_{pc}\) and \(\Delta \varepsilon_{cp}\)).

One of the advantages of the SRP rule is that it is relatively temperature independent (Halford, Hirschberg and Manson, 1973). The life relationships are governed by the four inelastic strain-ranges and not greatly affected by the temperature at which the strains are imposed. If particular strain-ranges are imposed at one temperature, the life will be similar to that when the same strain-ranges of the same type are applied at another temperature. This does not mean that life is independent of temperature because a given imposed load will produce different strains at a different temperature and a given strain will be partitioned differently depending on the temperature. As indicated by Manson (1973), the predicted results by both SRPs, one based on the linear damage rule in Equation (10) and the other based on the interaction damage rule in Equation (11), agreed well with the experimental results. The question of which damage rule yields more accurate predictions is still open.

Material data for TMF life prediction, comprises four baseline relationships between the partitioned strain-ranges and cyclic life on the particular material, and is normally obtained experimentally. This life data (cycles to failure vs. strain-range) for each of the PP, CC, PC and CP cycle types, is fitted to a Coffin-Manson equation.
To apply the SRP model to a particular life prediction, the inelastic strain must be partitioned into PP, CC, and CP or PC components. It is not easy to partition the inelastic strain experimentally. To overcome this difficulty, Manson, Halford and Nachtigall (1975) adopted the step-stress method. Stress is held constant at different points around the hysteresis loops, and steady-stress creep rates determined. Rates are then integrated throughout the period of one cycle to obtain the creep strain. However, the step-stress method is still regarded as difficult to apply, and further simplifications have been proposed, for example the simplified step-stress method and the loop inversion method (Nitta and Kuwabara 1988). The simplified method assumes that creep is negligible below a specified threshold temperature, and above this temperature creep strain rate is a linear function of time. The material creep strain is evaluated only during a heating period while the temperature exceeds the threshold, and can be expressed as follows:

\[ \Delta \varepsilon_{cp} = \frac{C \sigma_{T_{\text{max}}}^n t_1 (T_{\text{max}} - T_c)}{4(T_{\text{max}} - T_{\text{min}})} \]  

(12)

\( T_{\text{max}} \) and \( T_{\text{min}} \) are the maximum and minimum temperatures respectively, \( \sigma_{T_{\text{max}}} \) is the stress at \( T_{\text{max}} \), and \( T_c \) is the specified threshold temperature (taken to be 500 °C for low-alloy steels and 550 °C for austenitic steels by Nitta and Kuwabara). \( t_1 \) is time for one cycle without a hold time, and \( C \) and \( n \) are constants.

Since the simplified step-stress method may yield non-conservative results, Kuwabara et al. (1988) proposed another alternative, the loop inversion method. Figure 3 shows the principle of the loop inversion method, which assumes that the creep strain is generated only during a heating period. The cooling portion of the cycle is simply the inverse of the heating part of the cycle.

To obtain the four unique partitioned strain-range vs. cyclic life relationships, four separate PP, PC, CC and CP creep-fatigue tests have to be done (refer to Figs 2 and 3 of Halford, Hirschberg and Manson, 1973). Results of these tests are fitted to the Coffin-Manson Equation (6), and can be expressed as:

\[ \Delta \varepsilon_{pp} = A_{pp} (N_{pp})^{C_{pp}} \]  

(13)

\[ \Delta \varepsilon_{pc} = A_{pc} (N_{pc})^{C_{pc}} \]  

(14)

\[ \Delta \varepsilon_{cp} = A_{cp} (N_{cp})^{C_{cp}} \]  

(15)

\[ \Delta \varepsilon_{cc} = A_{cc} (N_{cc})^{C_{cc}} \]  

(16)

The coefficients A and exponents C are experimentally determined material constants. For some materials typically used for engine hot-section components, coefficients are published in the literature and these are collected in the Appendix. The data have been acquired for specific material categories under particular testing regimes, and strictly should be used for life prediction only under the conditions for which they have been
To determine the TMF life of engine components using the SRP model, the procedure is summarised as follows: Firstly, based on the cyclic stress vs. strain response, obtain the partitioned inelastic strain-ranges, $\Delta \varepsilon_{pp}$, $\Delta \varepsilon_{pc}$, $\Delta \varepsilon_{cp}$ and $\Delta \varepsilon_{cc}$, and then the corresponding strain fractions, $F_{pp}$, $F_{pc}$, $F_{cp}$ and $F_{cc}$. Secondly, from the total inelastic strain-range, solve for $N_{pp}$, $N_{pc}$, $N_{cp}$ and $N_{cc}$ using Equations (13) to (16). Finally, determine the TMF lives by the interaction damage rule Equation (11). The lives determined in the above steps are for a theoretical zero mean stress condition. Halford and Nachtigall (1979) modified the SRP model to incorporate the effect of mean stresses.

The SRP model has been extensively applied in the nuclear and aerospace industries. The frequency, hold time and temperature effects are built into the SRP model since these variables are implicit in the stress-strain hysteresis loop. However, it is important to recognise the limitations of the SRP method to make the best use of it.

1. The model is not applicable to non-ductile materials since the inelastic strain is too small to be determined correctly. Moreover, these materials may undergo metallurgical changes in the temperature range of their application. In this case the SRP method suffers from severe limitations (Del Puglia and Manfredi, 1979).

2. Although SRP works well for many high-ductility materials, it is not successful for nickel-base superalloys which are common in aircraft engines. Nitta et al. (1983) found limitations when applying SRP to a variety of materials.
3. The SRP model does not take into account the environmental attack caused by oxidation, and may over-predict the life (Bernstein 1982).

4. For TMF cycles, even though Halford and Manson (1976) proposed the step-stress method and Nitta and Kuwabara (1988) developed a simplified step-stress method and the loop inversion method, it is still difficult to partition the inelastic strain experimentally.

3.5 Total Strain Version of Strain-Range Partitioning

As mentioned above, the SRP model was formulated on an inelastic strain-range basis. It has limited capability in the low-strain, long-life regime, where the inelastic strains are too small to be detected experimentally or predicted accurately by analytical methods. Thus Halford and Saltsman (1983, 1988) extended SRP to the Total Strain version of Strain-Range Partitioning (TS-SRP). TS-SRP takes account of not only the damage due to the inelastic strain-range like SRP, but also the damage resulting from the elastic strain-range.

In TS-SRP, the various elastic strain-range (PP, PC, CP and CC) versus life relations are assumed to follow a power law, parallel to the corresponding PP elastic life line on a log-log plot. Similarly it is assumed that on a log-log plot the various inelastic strain-range versus life relations are parallel to the PP inelastic line. Hence the failure behaviour is given by:

\[ \Delta \varepsilon_{el} = B (N_f)^b \]  
\[ \Delta \varepsilon_{in} = C' (N_f)^c \]

and it can be shown that for the modified SRP Equations (11), (13)-(16) that:

\[ C' = \left[ \sum F_{ij} (A_{ij})^{1/c} \right]^c \]  \hspace{1cm} (19)

F_{ij} are the previously defined strain-range fractions associated with PP, PC, CP and CC, and the coefficients A_{ij} are those of Equations (13)-(16). From the parallelism assumption (shown in Fig. 4), b and c are constants independent of cycle time and waveshape. B is the intercept of the elastic strain-range versus life relation, and C’ is the intercept of the equivalent inelastic line for combined creep-fatigue cycles. Both B and C’ are cycle time and waveshape dependent.

The total strain-range is the sum of the elastic strain-range and the inelastic strain-range, so the total strain-range versus life equation can then be obtained as the following:

\[ \Delta \varepsilon_T = \Delta \varepsilon_{el} + \Delta \varepsilon_{in} = B (N_f)^b + C' (N_f)^c \]  \hspace{1cm} (20)
To apply this TS-SRP life prediction model the constants $b$, $c$, $B$ and $C'$ in Equation (20) must be determined. Flow behaviour (i.e. the cyclic stress-strain curve which is the locus of reversal points of stabilised hysteresis loops) offers an additional means of gaining data for TS-SRP. By eliminating $N_f$ from Equations (17) and (18), a relation between elastic and inelastic strain-ranges can be derived:

$$\Delta \varepsilon_{el} = B \left( \frac{\Delta \varepsilon_{in}}{C'} \right)^{b/c} \tag{21}$$

This can be compared with a flow law (determined by experiment or by constitutive modelling):

$$\Delta \varepsilon_{el} = \frac{\Delta \sigma}{E} = K (\Delta \varepsilon_{in})^n \tag{22}$$

$\Delta \sigma$ is the stress range, $E$ is Young's modulus, $n$ is the cyclic strain-hardening exponent and $K$ is the corresponding hardening coefficient. Equating the failure behaviour in Equation (21) and the flow behaviour in Equation (22) gives:

$$n = \frac{b}{c} \tag{23}$$

$$B = K (C')^n \tag{24}$$

The constants $b$ and $c$ are normally determined from PP failure tests, conducted in the high strain-range regime so that test times can be reduced and costs minimised. But obtaining an accurate value for $b$ is difficult so that $n$ may not exactly equal $b/c$. Thus, for better substantiation of $n$ additional PP flow tests can be conducted when the specimen is cycled for long enough for material shakedown (Saltsman and Halford 1989). The constants $B$ and $C'$ may be obtained by one of the three following methods depending upon the type of material information available.
(1) Constitutive flow modelling. Using an advanced cyclic constitutive flow model for which material constants are available, the accurate stress-strain hysteresis loop can be calculated numerically. Details of the elastic and inelastic strains are thus obtained and partitioned, and the strain fractions $F_{ij}$ calculated, so the constant $C'$ can be determined from Equation (19). The cyclic strain-hardening coefficient $K$ and exponent $n$ are found from the flow relation generated numerically by the constitutive model, and the elastic line intercept $B$ can be calculated using Equation (24).

(2) Failure testing. If a constitutive model is not available, failure tests can be used to obtain inelastic strain-range versus life relations and elastic strain-range versus life relation. Intercepts $C'$ and $B$ can then be determined by using the empirical approach. The failure tests should be performed at lower strain-ranges to reduce extrapolation errors, which means that testing time and costs will increase.

(3) Flow testing. Since tests to failure at lower strain-ranges are very time consuming and expensive, flow tests for creep-fatigue cycles can be conducted instead. Flow testing requires only that cyclic conditions be stabilised and testing need not be extended to failure, so lower strain-ranges can be tested. A flow equation obtained from the test data can then be used to calculate the intercept $B$ using Equation (24).

Method (1) is the most economical procedure. It is promising and will become more attractive as numerical constitutive modelling continues to be improved. In contrast, method (2) is the most expensive because lengthy failure tests have to be performed at lower strain-ranges. Method (3) is intermediate; flow testing is performed in the low strain regime but there is no need to prolong the test to failure.

### 3.6 Strain Energy Partitioning

He et al. (1983) synthesised strain-range partitioning and the Ostergren's damage function model and developed Strain Energy Partitioning (SEP). SEP assumes that the work done upon the material by the external forces causes fatigue damage. Like SRP, SEP has four partitioned strain energy vs. cyclic life relationships as follows.

\[
N_{pp} = A_{pp} \left( \Delta U_{pp} \right)^{C_{pp}}
\]
\[
N_{pc} = A_{pc} \left( \Delta U_{pc} \right)^{C_{pc}}
\]
\[
N_{cp} = A_{cp} \left( \Delta U_{cp} \right)^{C_{cp}}
\]
\[
N_{cc} = A_{cc} \left( \Delta U_{cc} \right)^{C_{cc}}
\]

where partitioned strain energy $\Delta U_{ij} = (\sigma_T \Delta \varepsilon_{ij})$, $\sigma_T$ is tensile peak stress, and $A_{ij}$ and $C_{ij}$ are material constants. Analogously to the FS model, only tensile stress is considered to produce microcrack growth and fatigue damage. Since SEP includes both inelastic
strain range and a stress value, it has the capacity to give better predictions for a wider range of materials of both low and high ductility.

The SEP model was assessed with experimental data for several turbine engine materials, such as, 1Cr-18Ni-9Ti for casing, GH36 and GH33A for turbine disks. Duan et al. (1988) compared SEP predictions with SRP and FS predictions, and found that only the SEP model was able to correlate the test data within the scatter band factor of two for all three materials. So far, the SEP model has not received much attention although it appears to be very promising. To make it more generally useful will require application and evaluation over a much wider range of materials and conditions, and possibly further refinement of the model.

4. Evaluation of the Models

Six TMF life prediction models, DS, FS, DE, SRP, TS-SRP and SEP, have been presented. Various papers indicate that it is not possible to conclude that one model is invariably better than the others. Research results from different institutes over the world come up with differing conclusions regarding which life prediction model is best. The validity of any one prediction model has to be assessed for a specific material and for a particular service application. Nevertheless, some researchers have compared the applicability of different models, although comparison is restricted because the data have been obtained for specific groups of materials under limited ranges of strain and temperature.

Comparing the life prediction models, Leven (1973) and Batte (1983) applied the DS, FS and SRP models to low alloy ferritic steels under conditions where the hysteresis loop is simple and well-defined, the material does not show significant cyclic softening or hardening, and the failure mode is not dominated by creep fracture. Under these so-called “favourable” conditions, they found that the models could predict life within a factor of two. They also pointed out if the conditions are less favourable, or there is a significant involvement of creep fracture in the failure process, some models can produce poor prediction.

For nickel-base superalloys, Bernstein (1982) evaluated the SRP, FS and FS variations, and concluded that no model is able to correlate the fatigue life of Rene 95 at 650 °C within the targeted scatter band of two because the models ignore mean stress and environmental effects. The FS model and its variations were able to correlate the baseline data within a factor of 3.6, while the SRP model correlates these data to a scatter band larger than five. But Nazmy (1988) used the FS, FS variations and SRP models to correlate high-temperature low-cycle fatigue data of MA 754 nickel-base alloy, and found that all three models were able to correlate the data within a scatter band factor of two. Testing was conducted in air at 850 °C. Nazmy and Wuthrich (1984) also compared the SRP and FS models for life prediction with Inconel 738 and
decided in favour of the SRP model. For the same alloy, Marchionni et al. (1986) obtained similar results.

Ellison and Zamily (1994) assessed TMF lives of 1CrMoV steel in the temperature range 315 to 565 °C, and two different batches of 316 stainless steel in the temperature range 400 to 625 °C. The DS, DE and modified SRP life prediction models were used in the study. Both the original and modified SRP models were able to predict lives within a factor of two. The DE model was not as accurate but at least yielded conservative life predictions for both steels. The DS model was not adequate to predict life within a factor of two and was also non-conservative, particularly for the 1CrMoV steel.

To validate the SEP model, Duan et al. (1988) compared the SEP model with the SRP and FS models for three turbine engine materials, 1Cr-18Ni-9Ti for casings, GH36 and GH33A for turbine disks. Tests were conducted in air at temperatures of 600 °C for 1Cr-18Ni-9Ti and 650 °C for GH36 and GH33A. The studies found that for these three materials, only the SEP model was able to correlate the test data within the scatter band factor of two. It was also stated that the higher the strength of the tested material, the better the accuracy of the SEP model compared with the SRP and FS models, mainly because the stress variable included in the SEP model has a more important role for high-strength and low-ductility materials.

All of the life prediction models are able to predict life only with limited accuracy. It has not been possible to identify the most satisfactory model for all alloys under a varying range of service conditions. It is almost certain that the DS model, with its relative ease of application over a wide range of conditions and with ready availability of much support data, will be the mainstay of life prediction models in design codes for some time. The SRP and TS-SRP models have attracted the most research effort and are attractive life prediction techniques since their generic life component equations are nominally temperature insensitive.

5. Conclusions

A review has been made of models for predicting TMF crack initiation life. A large number of models are available, and a limited number of the more popular models are described in detail. All the models reviewed are able to predict life with acceptable accuracy within a certain envelope of material and test conditions, but no model can consistently produce accurate life predictions for all materials under different service conditions. Therefore a prediction model should be used only when it has been validated for the material category and expected environmental conditions. If data is not available, predicted lifetimes should be validated against laboratory test data or by in-service monitoring.

There are various reasons for the shortcomings of the models, such as failure to model stress-relaxation and creep characteristics caused by strain softening or hardening. Life
prediction models available today are not based entirely on sound physical principles. Rather they are phenomenological in nature, involving empirical constants that are material-dependent and often difficult to evaluate.

Although the SRP and TS-SRP models have attracted great research effort, all the current design codes and rules are still based on the earlier linear Damage Summation model. Further research on life prediction models is needed, and long-term validation and analysis will be required to make the new models more generally acceptable.

The review provides a material database for the SRP model for selected engine hot section materials. To extend the usefulness of the database for engine life management, a search for further relevant material data should be continued in the open literature and in any unpublished data that become available.

6. Areas for Further Research

This review on TMF life prediction has indicated that there are still many limitations associated with each of the models. Better understanding is needed in the following areas:

1. The development of a comprehensive life prediction model is a long-term goal. Prospects of success in this respect lie in improving the depth of understanding of the TMF damage process and in determining the damage parameters without need for excessive testing. Refinements in damage modelling and difficulty of implementation (both in extent of material testing and complexity of problem analysis) need to be balanced.

2. It is necessary to develop some new concepts of damage modelling to handle complex TMF loading, such as triaxial loading and non-proportional loading.

3. Extending the formulation of the very popular SRP life prediction model to incorporate the effect of environment is most desirable. Some modification has been made to SRP, but it has not so far led to a significant improvement in the life predictions.

4. For the effective application of TS-SRP, development of appropriate material constitutive models is essential to characterise the behaviour of the alloys under study and more readily obtain some of the constants in the TS-SRP equations.

5. Although SEP appears to be very promising, it has not attracted much interest. Further application and evaluation of the SEP model over a much wider range of materials and conditions are required, which could lead to refinement of the model.
7. References


Appendix: Material Database for SRP

This appendix contains material data, comprising the parameters used in the Strain-Range Partitioning model for life prediction. The data have been acquired under particular testing regimes, so before making use of them, care should be taken to ensure that they are applicable. Detailed references are supplied to help trace these data. For different conditions, data should be developed from appropriate tests and analyses. Material parameters are given in Table 1, corresponding to terms in the SRP life equations (13) - (16) in this report, repeated below.

\[
\begin{align*}
\Delta \varepsilon_{pp} &= A_{pp} (N_{pp})^{C_{pp}} \\
\Delta \varepsilon_{pc} &= A_{pc} (N_{pc})^{C_{pc}} \\
\Delta \varepsilon_{cp} &= A_{cp} (N_{cp})^{C_{cp}} \\
\Delta \varepsilon_{cc} &= A_{cc} (N_{cc})^{C_{cc}}
\end{align*}
\]  

(13) (14) (15) (16)

Table 1. Material data for the SRP model

<table>
<thead>
<tr>
<th>Alloy</th>
<th>$A_{pp}$</th>
<th>$C_{pp}$</th>
<th>$A_{pc}$</th>
<th>$C_{pc}$</th>
<th>$A_{cp}$</th>
<th>$C_{cp}$</th>
<th>$A_{cc}$</th>
<th>$C_{cc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF2-1DA</td>
<td>0.083</td>
<td>-0.600</td>
<td>0.754</td>
<td>-1.090</td>
<td>0.056</td>
<td>-0.600</td>
<td>0.088</td>
<td>-0.600</td>
</tr>
<tr>
<td>Hastelloy X</td>
<td>0.544</td>
<td>-0.680</td>
<td>0.097</td>
<td>-0.510</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>MAR-M509</td>
<td>1.866</td>
<td>-0.403</td>
<td>1.786</td>
<td>-0.252</td>
<td>1.746</td>
<td>-0.242</td>
<td>10.955</td>
<td>-0.893</td>
</tr>
<tr>
<td>Rene 95</td>
<td>0.795</td>
<td>-0.909</td>
<td>0.378</td>
<td>-1.149</td>
<td>1.464</td>
<td>-1.134</td>
<td>0.183</td>
<td>-0.833</td>
</tr>
<tr>
<td>IN 939</td>
<td>0.333</td>
<td>-0.850</td>
<td>0.333</td>
<td>-0.850</td>
<td>0.048</td>
<td>-0.600</td>
<td>0.048</td>
<td>-0.600</td>
</tr>
<tr>
<td>MA 754</td>
<td>0.990</td>
<td>-1.818</td>
<td>0.100</td>
<td>-2.857</td>
<td>0.005</td>
<td>-2.941</td>
<td>0.100</td>
<td>-5.882</td>
</tr>
<tr>
<td>IN 738</td>
<td>0.064</td>
<td>-0.630</td>
<td>0.036</td>
<td>-0.530</td>
<td>0.141</td>
<td>-0.850</td>
<td>0.253</td>
<td>-0.990</td>
</tr>
</tbody>
</table>

1. AF2-1DA

Nickel-base superalloy AF2-1DA is an advanced gas turbine disk alloy for high-performance military aircraft engines. The material is a powder metallurgy alloy. Its elastic modulus is 25,000 ksi, yield strength (0.2% offset) 123.4 ksi, and ultimate tensile strength 163.7 ksi. The heat treatment is described in the reference:


2. Hastelloy X

A fine grained nickel base superalloy, Hastelloy X is a material commonly used in sheet form for gas turbine combustor liners and similar parts. It has moderately good
strength characteristics up to nearly 900 °C. Temperature dependent material properties are given in Table 2.

Table 2. Temperature-dependent material properties

<table>
<thead>
<tr>
<th>Temp. °C</th>
<th>$E \times 10^5$ MPa</th>
<th>$\nu$</th>
<th>$\alpha \times 10^5 / °C$</th>
<th>$\sigma_{ys}$ MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>2.05</td>
<td>0.320</td>
<td>1.42</td>
<td>380</td>
</tr>
<tr>
<td>505</td>
<td>1.70</td>
<td>0.326</td>
<td>1.49</td>
<td>316</td>
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<tr>
<td>649</td>
<td>1.61</td>
<td>0.334</td>
<td>1.54</td>
<td>304</td>
</tr>
<tr>
<td>760</td>
<td>1.52</td>
<td>0.339</td>
<td>1.58</td>
<td>263</td>
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<tr>
<td>871</td>
<td>1.37</td>
<td>0.345</td>
<td>1.62</td>
<td>101</td>
</tr>
<tr>
<td>980</td>
<td>1.24</td>
<td>0.351</td>
<td>1.66</td>
<td>49</td>
</tr>
</tbody>
</table>


3. MAR-M 509
MAR-M 509 is a cast cobalt superalloy used in nozzle guide vanes. The composition of the material is 0.59 C, 11.0 Ni, 23.2 Cr, 6.55 W, 3.31 Ta, 0.30 Zr, 0.22 Ti, 0.17 Fe, 0.008 B, 0.005 P, 0.003 Si, balance Co.


4. Rene 95
Thermo-mechanically processed, cast and wrought Rene 95 is an advanced nickel-base superalloy which has high tensile strength, excellent creep resistance, and good ductility up to 650 °C. It has been used in jet engine turbine disks. The principal alloying elements (wt. %) are 14 Cr, 8 Co, 3.6 W, 3.6 Al, 3.5 Mo, 3.5 Cb, 2.5 Ti, and 0.15 C.


5. IN 939
IN 939 is a cast nickel-base superalloy which is used for gas turbine blades. Its composition (wt. %) is: 0.145 C, 0.15 Si, 0.003 S, 1.93 Al, 0.009 B, 0.98 Cb, 19.10 Co, 22.55 Cr, 0.05 Fe, 1.38 Ta, 3.67 Ti, 2.06 W, 0.105 Zr, balance Ni.

International Conference on Advances in Life Prediction Methods, New York, pp. 131-141, 1983.

6. **MA 754**

Inconel MA 754 is a nickel-base superalloy used commonly for nozzle guide vanes. Composition (wt. %) is: 20.32 Cr, 0.31 Al, 0.4 Ti, 0.98 Fe, 0.05 C, 0.01 S, 0.57 Y₂O₃, balance Ni.


7. **IN 738**

IN 738 is a cast nickel-base superalloy used normally for turbine blades and nozzle guide vanes. The composition (wt. %) is: 0.09 C, 8.25 Co, 15.95 Cr, 0.011 B, 1.6 Ta, 3.5 Al, 3.45 Ti, 0.7 Nb, 2.48 W, 0.5 Zr, 1.62 Mo, balance Ni.

**Thermo-Mechanical Fatigue Life Prediction: A Critical Review**

**Abstract**

Improved prediction methods for thermo-mechanical fatigue life will assist in reducing life cycle costs and increasing the availability of the hot-section components in aircraft engines. Literature on thermo-mechanical fatigue life assessment is reviewed in this report, with an emphasis on the life prediction models applied in aircraft engines. Successful areas of application of these life prediction models are addressed as well as their limitations. Published quantitative thermo-mechanical fatigue life data for selected hot section materials is also summarised. The review concludes by indicating areas where knowledge is deficient and where further research would be most beneficial.