Reliability Block Diagram

Combinatorial Models
RBD for Series Systems
RBD for Parallel Systems
Non-State-Space Modeling Techniques TAXONOMY

- Non-State-Space method
  - Performance models
    - Queuing models
  - Dependability models
    - Fault Tree models
    - Reliability Block Diagram models
    - Reliability Graphs
Combinatorial Models

- Use **probabilistic techniques** to enumerate the different ways in which a system can remain operational.
- The reliability of a system is **derived** in terms of the reliabilities of the **individual components** of the system (thus the term combinatorial).
- Examples: one-component system, two-component system, …
Complexity Concerns

For a system consisting of \( n \) components

Every component can be in one of the two conditions: working or failed

How many possible combinations of the status of these \( n \) components?

How do you calculate the reliability of the system given the probability of each component being working (or failed)?
What can be done dealing with the complexity?

During model construction:
- Need a more intelligent way to describe the system’s failure behavior
- Series and parallel structures

During model solution:
- Need more efficient ways of calculations, rather than counting individual probabilities
“Structured” Combinatorial models

- **Reliability block diagrams**, Fault trees and Reliability graphs
  - Integrate certain probability events into a module
  - Organize the modules in a “structured” way, according to the effects of each module’s failure
  - Commonly used in reliability, availability, or safety assessment
  - These model types are similar in that they capture conditions that make a system fail in terms of the structural relationships between the system components.
Features

- Combinatorial modeling techniques like RBDs and FTs are easy to use
- Assuming **statistical independence**
  - Failures independence
  - Repairs independence
- Each component can have attached to it
  - A probability of failure
  - A failure rate
  - A distribution of time to failure
  - Steady-state and instantaneous unavailability
Features continue

- Easy specification,
- Fast computation
  - Relatively good algorithms are available for solving such models so that 100 component systems can be handled computationally (consider the case where you need to handle $2^{100}$ probability events)
Series Systems

- A system that contains no redundancy
- Each component of the system is needed to make the system function correctly
- If any one of the components fails, the system fails
- Example:
RDB Example: Series System

System Block Diagram
RDB example: Series System

- Reliability Block Diagram

Monitor ➔ Processor ➔ Keyboard
RDB Example: Reliability Calculation for Series RBD

- Let $\lambda_1$ be the failure rate for Monitor
- Assume exponential distribution for the failures
- $R_{\text{monitor}}(t) = e^{-\lambda_1 \cdot t}$
- Similarly, $R_{\text{processor}}(t) = e^{-\lambda_2 \cdot t}$ and $R_{\text{keyboard}}(t) = e^{-\lambda_3 \cdot t}$
- $R_{\text{system}}(t) = R_{\text{monitor}}(t) \cdot R_{\text{processor}}(t) \cdot R_{\text{keyboard}}(t)$
- When exponential failure distribution is assumed, the failure rate of a series system is the sum of individual components’ failure rates
RDB Example: Availability Calculation

- Let $\lambda_1$ be the failure rate for Monitor, and $\mu_1$ be the repair rate for the Monitor.
- Exponential distributions are assumed.
- $A_{SS-Monitor} =$
- $A_{SS-processor} =$
- $A_{SS-keyboard} =$
- $A_{SS-system} =$
Something you must keep in mind when applying RBD

- Failure/Repair Dependencies are often present
- RBDs, Fault Trees cannot easily handle the dependency such as
  - Event-dependent failure
  - Shared repair
Hierarchical Composition Method

Given a detailed description of a system, too many components are displayed, which makes the modeling task difficult which creates unnecessary complexity.

Abstract the detailed description into a higher level description – hierarchical composition method.
Homework Exercise: A simple Aircraft Control System

- Use the system block diagram given in the handout, construct the corresponding RBD
- Abstract the system block diagram into a higher level block diagram
- From the higher level system block diagram, construct the corresponding RBD
- Each block in the higher level RBD has its own RBD underneath
Failure rates

- $\lambda_{\text{sensor}} = 1 \times 10^{-6}$ failures per hour
- $\lambda_{\text{actuator}} = 1 \times 10^{-5}$ failures per hour
- $\lambda_{\text{computer}} = 4 \times 10^{-4}$ failures per hour
- $\lambda_{\text{bus}} = 1 \times 10^{-6}$ failures per hour
Homework (due next Tue.)

- Plot the system reliability as a function of
  (1) The failure rate of “Computer”
  (2) The failure rate of “Actuator”
  (3) The failure rate of “Bus”
  (4) The failure rate of “Sensor”
Parallel Systems

- **Basic** parallel system: only one of the $N$ identical components is required for the system to function

- Example:
RDB Example: Basic Parallel System

System Block Diagram
RDB example: Parallel System

Reliability Block Diagram

- Keyboard
- Processor
- Monitor
- Keyboard
- Processor
- Monitor
RDB using Hierarchical Composition/Decomposition

On the higher level (overall system level)

On the “Computer” level
Reliability Calculation for Basic Parallel RBD

- The “Unreliability” of the parallel system can be computed as the probability that all $N$ components fail.

- Assume all $N$ components are having the same failure rate $\lambda$, and the probability that a component is failed at time $t$ is $P_{\text{fail}}(t)$.

- \[ 1 - R_{\text{parallel}}(t) = \prod_{i=1}^{N} P_{\text{fail}}(t) \]

- If exponential distribution is used for $P_{\text{fail}}(t)$, what would be the equation for $R_{\text{parallel}}(t)$?
Independence Assumption

- Where in the above equation that the independence assumption is made?
- Just to remind you…

- **Failure/Repair Dependencies are often present**
- **RBD cannot easily handle the dependency such as**
  - Event-dependent failure
  - Shared repair (when availability is considered)
Availability Calculation

- Let $\lambda_1$ be the failure rate for Monitor, and $\mu_1$ be the repair rate for the Monitor.
- Exponential distributions are assumed.
- $A_{SS-Monitor} =$
- $A_{SS-processor} =$
- $A_{SS-keyboard} =$
- $A_{SS-1of2-parallel-system} =$
Comparison

- $\lambda_{\text{monitor}} = 1 \times 10^{-4}$ failures per hour
- $\lambda_{\text{processor}} = 1 \times 10^{-5}$ failures per hour
- $\lambda_{\text{keyboard}} = 4 \times 10^{-4}$ failures per hour
- $\mu = 2$ repair per hour for all components
- For series system, $A_{SS}$ is

- For parallel system (with 1:2 redundancy), $A_{SS}$ is
Parallel/Series System

Processor 1      Keyboard 1      Monitor 1
  |             |             |
  Bus 1         Bus 2         Bus 2

Processor 2      Keyboard 2      Monitor 2

What is the corresponding RBD?
Corresponding RBD

Assuming Buses are perfect

Monitor

Processor

Keyboard

Monitor

Processor

Keyboard

Monitor

Processor

Keyboard

Compare to the RBD shown before, what is the difference?
### Numerical Comparison (1)

<table>
<thead>
<tr>
<th>Component</th>
<th>$P_w$</th>
<th>$P_f$</th>
<th>$P_w$ (1 of 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monitor</td>
<td>0.99</td>
<td>0.01</td>
<td>0.9999</td>
</tr>
<tr>
<td>Keyboard</td>
<td>0.9</td>
<td>0.1</td>
<td>0.99</td>
</tr>
<tr>
<td>Processor</td>
<td>0.999</td>
<td>0.001</td>
<td>0.999999</td>
</tr>
</tbody>
</table>

$P_{system-w} = 0.98990001$
Numerical Comparison (2)

<table>
<thead>
<tr>
<th>Component</th>
<th>$P_w$</th>
<th>$P_f$</th>
<th>$P_{w\text{-single}}$</th>
<th>$P_{\text{system-w}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monitor</td>
<td>0.99</td>
<td>0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Keyboard</td>
<td>0.9</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Processor</td>
<td>0.999</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$P_{w\text{-single}} = 0.890109$  
$P_{\text{system-w}} = 0.987923968$
Non-Identical Parallel Systems

What if the two components (for processor, or for monitor, or keyboard) have different failure rates? How are you going to calculate the reliability?
N Modular Redundancy

- $M$ of $N$ System
  - $M$ of the total of $N$ identical modules are required to function
  - TMR is one example, where $M$ is 2 and $N$ is 3
RBD for TMR

Module 1

Module 2

Module 3

Voter

2:3
Reliability Calculation for TMR

Cases for the TMR to be working:
• all of the 3 modules are working
• any 2 modules are working, and 1 module is failed

Look at it from another way:
Cases for the TMR to be failed
• all 3 modules are failed
• any one module is working, however, the rest 2 are not working

Remember, the voter is a Single-Point-Of-Failure

<table>
<thead>
<tr>
<th>Module</th>
<th>voter</th>
<th>TMR</th>
<th>System Pw</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.999</td>
<td>0.999</td>
<td>0.999997</td>
<td>0.998997005</td>
</tr>
</tbody>
</table>

2 : 3
Reliability of the TMR system as a function of Rel of the Module & Rel of the Voter

- changing Rel of Voter
- changing Rel of Module

Rel (TMR System)

Rel of individual component (voter or module)
WAAS System Block Diagram

http://gps.faa.gov/Programs/index.htm
Construct the RBD for the WAAS System Block Diagram

- Use Excel
- Use Sharpe to implement your model