Case 6
Modeling Horizontal Strip Casting of Copper Alloys

A process has been proposed to continuously cast copper alloys strips by pouring molten metal from a specially designed tundish through a slot and onto the surface of a water cooled, moving substrate. It has been recognized that the process involves an interplay between fluid flow phenomena and heat transfer solidification processes. A prototype of the system has been built and a number of experimental trials have been undertaken. However, it is felt that mathematical modeling can provide an improved understanding of the process as well as quantitative estimates of the relationships existing among the various process parameters.

A mathematical model of the process then requires:

- the statement of mass conservation (equation of continuity),
- the statement of momentum conservation (equation of motion)
- the statement of energy conservation (energy equation), as well as
- appropriate sets of boundary conditions for the above.

Some of the key assumptions introduced to make the problem tractable are as follows:

- laminar flow is assumed,
- since the metal flows out of a narrow slot, restriction to two dimensions seems appropriate,
- the fluid is assumed Newtonian and flow is assumed laminar,
- the constitutive flow behavior of the solidfying material is handled by introducing a temperature dependent viscosity that grows exponentially as the fraction solidified approaches the value of one,
- values of heat transfer coefficients obtained from experimental data in the prototype are used.

After selecting a rectangular Cartesian system of coordinates, one proceeds to the statement of the equation of continuity

$$\nabla \cdot \mathbf{v} = 0$$

and the equation of motion

$$(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho g$$

The boundary conditions include no slip at solid boundaries and zero shear at free boundaries.

Concerning the thermal aspect of the problem, this requires statement of the energy equation

$$\mathbf{v} \cdot \nabla H = \nabla(k \nabla T)$$

By incorporating the latent heat into the definition of H the solidification process can be readily taken into account. The boundary conditions associated with the above include the temperature of the incoming melt as well as the heat extraction rate through the substrate and the heat losses (convection and radiation) through other solid surfaces and through the free surface.

Exact analytical solution of the above does not appear possible so one must resort to numerical methods. A modification of the standard Marker and Cell (MAC) method was implemented. In this method, the solution to the steady state problem is obtained by solving a transient problem until the steady state is reached. First, a rectangular mesh is first created. The pressure and the temperature are calculated at the mesh nodes and the velocity components are calculated at the midpoints between nodes (staggered mesh). The equation of motion is discretized using the finite difference method and the result is the following set of linked simultaneous algebraic equations for the velocity components u and v

$$\frac{\Delta u}{\Delta t} + u_{i+1/2,j}\frac{\Delta u}{\Delta x} + v_{i+1/2,j}\frac{\Delta u}{\Delta y} = -\frac{\Delta p}{\Delta x} + \mu\frac{\Delta}{\Delta x}(\frac{\Delta u}{\Delta x}) + \mu\frac{\Delta}{\Delta y}(\frac{\Delta u}{\Delta y})$$

$$\frac{\Delta v}{\Delta t}u_{i+1/2,j}\frac{\Delta v}{\Delta x} + v_{i+1/2,j}\frac{\Delta v}{\Delta y} = -\frac{\Delta p}{\Delta y} + \mu\frac{\Delta}{\Delta x}(\frac{\Delta v}{\Delta x})\mu\frac{\Delta}{\Delta y}(\frac{\Delta v}{\Delta y})$$

where all the finite differences, except the one for the time derivative, are obtained using second order accurate central difference formulae. A simple forward difference formula is used to approximate the time derivative.

The standard approach is to start with a guessed velocity field at $t = 0$ and then use the above equations explicitly to obtain values of u and v at $t = \Delta t$. However, since the above equations involve the pressure, one must first obtain values for it. The value are obtained by solving a discretized equation for the pressure obtained from the equation of continuity. The approach used in MAC is to construct a Poisson equation for then pressure in terms of the velocities. This equation is readily solved by iterative methods such as Gauss-Seidel or Successive over-relaxation (SOR). The discretized pressure equation is

$$\frac{\Delta}{\Delta x}(\frac{\Delta p}{\Delta x}) + \frac{\Delta}{\Delta y}(\frac{\Delta p}{\Delta y}) = -(\frac{\Delta}{\Delta x} + \frac{\Delta}{\Delta y}) \cdot [(\mathbf{v} \cdot \nabla)\mathbf{v}]$$

The discretized form of the energy equation is

$$\frac{\Delta H}{\Delta t} + u\frac{\Delta H}{\Delta x} + v\frac{\Delta H}{\Delta y} = \frac{\Delta}{\Delta x}(k\frac{\Delta T}{\Delta x}) + \frac{\Delta}{\Delta y}(k\frac{\Delta T}{\Delta y})$$

with centered differences used everywhere but in the time derivative, where a simple forward difference is used.

In sum, the computation of proceeds as follows:

- assume initial velocity and temperature fields,
- compute values of p for all nodes solving the pressure equation by iteration,
- obtain new values of u and v from the old ones and the freshly calculated values of p,
- obtain new values of the H and T from the old ones,
- to reach steady state, repeat the calculation above until the changes computed values of u, v, p, H and T are sufficiently small from one time level t, to the next $t + \Delta t$.

Slight modification of the standard MAC algorithm allows the treatment of the solidification process. The resulting algorithm was coded in FORTRAN, a number of verification and validation tests were performed and then it was used for production runs. A selected sample of results obtained is shown in the attached figures.