Comparative Study of a Method of Characteristics and Numerical Solution Approach for and Euler Based Flow Field Solution Of Supersonic Projectile Flight through Air

5 March 09

Keith Smith
A Proposal Submitted to the Graduate Faculty of Rensselaer Polytechnic Institute In Partial Fulfillment of the Requirements for the degree of MASTER OF MECHANICAL ENGINEERING

Author: /s/ K. D. Smith 1 March 09
Keith D. Smith

Advisor: 
Dr. David Tew

Department:
Dr. Ernesto Gutierrez-Miravete
1 ABSTRACT

This paper presents a comparative study of solution methods to estimate the flow field near a supersonic projectile in air. A Method of Characteristics (MOC) technique is used to solve the two-dimensional Euler equations for a generalized axi-symmetric projectile. The projectile is in a condition of supersonic steady state flight with an angle of attach of zero degrees. The results are compared to a closed form solution for the bow shock and trailing Prandtl-Meyer expansion, and to a Reynolds Averaged Navier-Stokes (RANS) flow field where fluid rotation and thermal dynamic heating is neglected (i.e., an Euler solution). The RANS solution is obtained using FLUENT® Software application. The results of this study showed <reserved>.

2 INTRODUCTION

The purpose of this paper is to illustrate the relative accuracy and stability of the Method of Characteristics technique applied to the computational fluid dynamics (CFD) problem of a generalized supersonic projectile travelling through air. The MOC results are compared to analytic results from compressible flow theory for oblique shock and Prandtl-Meyer expansions, and another numerical solution obtained from finite difference methods. Topic and Objective statements are presented below and the problem, approach and anticipated results are described in the following sections.

**Topic Statement:** Complete a comparative study of the MOC and a numerical approach for supersonic projectile flight through air using Fluent® software. The focus is on accurate implementation of the MOC.

**Objectives Statement:** The objective of this study is to evaluate the similarities (or differences) of resulting velocity (and pressure as time permits) fields obtained from a Method of Characteristics solution to the supersonic projectile flight problem. The study focuses on accurate representation of the tip shock and Prandtl-Meyer expansion regions of the flow field and will qualitatively discuss turbulence near the boundary layer and in the wake region. Quantitative discussion may be left as future work.

3 PROBLEM SUMMARY

A kinetic energy projectile is travelling through air at supersonic speed without spin. An effort will be made to choose a body and mach speed consistent with previous work to increase the potential for use of already published data. Sources for flow field data have not been identified at this time. However, several NACA and military articles offer standardized projectile shapes and experimentally confirmed drag and/or lift coefficients (Note: It is not the intent of this paper to confirm lift or drag though these parameters may be useful for model verification).
A single velocity will be taken in a region from mach 1.5 to 4.5 with zero angle of attack. An experimental illustration is presented in Figure 3-1. The figure is annotated with the features that are the focus of this paper, and points out features that are beyond the scope of this work. The following compressible flow assumptions may be employed in the solution:

- Isentropic (adiabatic reversible) – no heat added
- Perfect gas – no intermolecular consideration (near-Earth atmosphere)
- Inviscid flow – no friction
- Irrotational flow – rotational derivatives are zero
- Steady state conditions – time derivatives are zero

4 APPROACH

The MOC solution will be bounded by an analytical solution to establish expected position and strength of the bow shock and expansion regions, and a finite difference numerical solution. A brief description of each approach is presented below.

The analytical calculations will make use of symmetry about the projectile center axis to examine half of the flow field, and the simplifying assumptions described in section 3. The compressible flow equations for oblique shock and Prandtl-Meyer expansions will be applied to locate the (attached) bow shock and expansion wave regions. The results will be plotted using MatLab® software (script will be attached to paper as Appendix A).
The MOC solution will present the two-dimensional (2D) velocity (and pressure as time permits) field results near the projectile body. Symmetry will be applied about the projectile center axis as illustrated in Figure 4-1 to reduce solution area and simplify boundary conditions. The MOC calculations will solve conservation of mass, energy, momentum and equation of state as necessary to estimate the velocity field near the projectile. Details of the calculation will be carried out using MatLab® software (script will be attached to paper as Appendix B). MatLab® will also plot the resulting fields.

The numerical solution will use the same problem setup described above. However, these results will be obtained using available FLUENT® and Gambit® software. Since I am learning this software as part of the project, I would like access to the RPI application to begin tutorial session prior to the start of the paper.

Figure 4-1: Illustration of Numerical Solution Setup (with boundary conditions)[4]

5 ANTICIPATED RESULTS

The analytic and finite difference solutions for this problem have been calculated before and serve to envelope (in terms of computational assumptions and complexity) the MOC solution. The velocity data will produce at least 3 field figures (6 if pressure calculations are completed). Computational accuracy of the MOC calculations will be determined and compared to like terms from FLUENT® solutions.
6 SCHEDULE

The major milestones for this proposal are outlined in Table 6-1 below. To ensure timely completion, *I would like to begin the initial steps as soon as advisor and department head approval is granted for this proposal.*

<table>
<thead>
<tr>
<th>Milestone</th>
<th>Semester Week</th>
<th>Target Date</th>
<th>Required Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topic Selection</td>
<td>1</td>
<td>11 May 09</td>
<td>(in proposal)</td>
</tr>
<tr>
<td>Objectives Defined</td>
<td>1</td>
<td>11 May 09</td>
<td>(in proposal)</td>
</tr>
<tr>
<td>Complete Literature Search</td>
<td>3</td>
<td>25 May 09</td>
<td></td>
</tr>
<tr>
<td>Complete Preliminary Approach</td>
<td>5</td>
<td>8 June 09</td>
<td></td>
</tr>
<tr>
<td>Complete Analysis/Calculations</td>
<td>8</td>
<td>29 June 09</td>
<td></td>
</tr>
<tr>
<td>Complete Draft Document</td>
<td>9</td>
<td>6 July 09</td>
<td>10 July 09</td>
</tr>
<tr>
<td>Complete Review/Revisions</td>
<td>12</td>
<td>27 July 09</td>
<td></td>
</tr>
<tr>
<td>Complete Final Review/Approval</td>
<td>14</td>
<td>15 August 09</td>
<td>(exams end)</td>
</tr>
<tr>
<td>Close</td>
<td>15</td>
<td>24 August 09</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** The references listed herein have not been reviewed thoroughly for application to this paper. Substitutions may be made as the paper progresses.
7 REFERENCES


